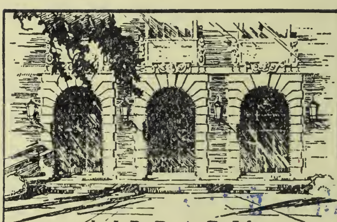

MANUAL OF THE SEXTANT.

MANUAL

OF

THE SEXTANT.

CONTAINING INSTRUCTIONS FOR ITS USE IN

DETERMINING TIME, LATITUDE, LONGITUDE, AND THE VARIATION OF THE COMPASS.

BY

CHARLES W. THOMPSON, F.R.G.S.

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JOHN BUMPUS, 350 OXFORD STREET,
BOOKSELLER BY SPECIAL APPOINTMENT TO
HER MAJESTY THE QUEEN.
1887.

a. c.
S. C. ~~Sextant.~~
" Compass

TO THE RIGHT HONOURABLE

LORD ABERDARE, G.C.B., F.R.S.

ONE OF THE VICE-PRESIDENTS

(AND LATE PRESIDENT)

OF THE ROYAL GEOGRAPHICAL SOCIETY,

THIS WORK IS, BY PERMISSION,

Respectfully Dedicated.

PREFACE.

This work is published with a view to meet the requirements of Explorers, Military Officers, Surveyors, and general travellers who may wish to fix by astronomical observations the positions of places they visit, and to afford to watchmakers and others living in localities devoid of telegraphic communication with an observatory, instruction as to the best means of obtaining with the Sextant accurate determinations of the *mean time* of remote localities. To Naval Officers employed in hydrographic operations on shore demanding greater precision than is required, or indeed attainable, in observations made at sea for the ordinary purposes of navigation, it is hoped that this Manual will prove of practical use.

Considerable pains have been taken to explain as simply and concisely as possible the construction, use, and capabilities of the instrument, while the methods of deducing the results of observations have been demonstrated by *examples* shewing each step of the computation in such a manner that the introduction of but few precepts has been found necessary to enable any one acquainted with the use of Logarithmic Tables to understand at once how

the observations should be reduced. These examples have been worked from actual observations taken specially for the purposes of this work, and have been carefully re-computed after being in type, so as to ensure their freedom from those errors which, however trivial in themselves, often cause perplexity to the reader.

The chief aim of the author has been to enable any person possessing a Nautical Almanac and Logarithmic Tables (even though he be entirely unacquainted with astronomical observations) not only to work with a sextant sufficiently well for purposes of navigation, but to obtain reliable determinations of latitude, longitude, &c., which shall, as regards accuracy, satisfy even the rigorous demands of modern geography. To this end the reader's attention has been drawn to the many sources of minute errors to which sextant observations are liable. The best methods of avoiding, eliminating, or diminishing them have been explicitly described. The remarks on the identification of the stars will prove serviceable to an observer who has not sufficient knowledge of the positions of the celestial bodies to recognize stars and planets suitable for observation, and who may not have access to star-maps or a celestial globe; while in the *definitions* which are given he will find much that will help him to a proper comprehension of the subjects dealt with, as well as formulæ which will serve to guide him in working out his computations.

The precision of the Tables is such as to render them applicable in the reduction of the best possible observa-

tions made with the sextant and artificial horizon, and they will be found self-explanatory, clear, and in such form as to give the greatest ease in practical working.

Woodcuts have been inserted where it was deemed necessary to illustrate the text.

While many excellent descriptions of the sextant and its use in nautical practice are to be found in works on Navigation and Nautical Astronomy, yet these are written more especially for instruction in observing at sea from the natural horizon, and omit much information which is essential to those employing the instrument on land, and working with an *artificial horizon*, in which latter circumstances a far greater degree of precision is attainable. It is therefore considered that the work now offered to the public will prove valuable to the class of readers to whom it is specially addressed.

CHARLES W. THOMPSON.

25*th July* 1887.

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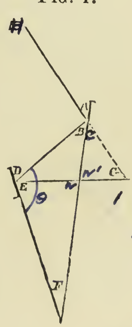
PRINCIPLE OF THE SEXTANT.

A RAY of light impinging on a plane reflecting surface, such as a perfectly even mirror, proceeds in a plane at right angles to this surface, and the angle of reflection is equal to the angle of incidence.

Fig. I.



When a ray is successively reflected from two plane surfaces, placed at right angles to a third plane, the angular deviation of the ray will be equal to twice the angle of inclination of the two surfaces to each other.

Let a ray of light H be successively reflected from the two mirrors A B and D E; the ray's deviation will be the angle G, the inclination of the mirrors the angle F.

1 $\theta = 180° - (B + F)$
$D = 180° - \theta$
$= 180° - 180° + (B + F) = B + F$

The angle of reflection B = the angle of incidence A.

The opposite angles A and C are equal.

$\measuredangle A = \measuredangle B = \measuredangle C$

The angle of reflection E = the angle of incidence D.

$$\text{Then } D = B + F = C + F, \text{(1)}$$

$$\text{and } E + F = C + G, \text{(2) or } D + F = C + G.$$

2. $\measuredangle N = \measuredangle N'$
$N = 180 - (E + F) = 180 - (C + G)$. ∴ $E + F = C + G$

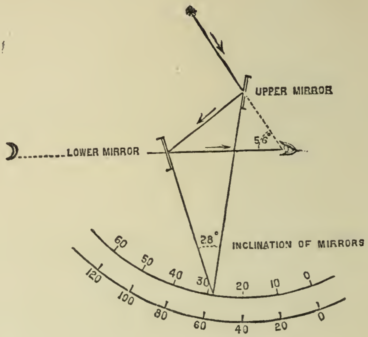
The difference of these two equations gives

$$G = 2\ F.$$

Suppose two mirrors inclined at an angle of 28°, as in the following figure, a ray of light falling from the star on to the upper mirror is reflected to the lower one, and thence

to the eye; then the total deviation of the ray is 56°, twice the inclination of the mirrors.

Fig. 2.



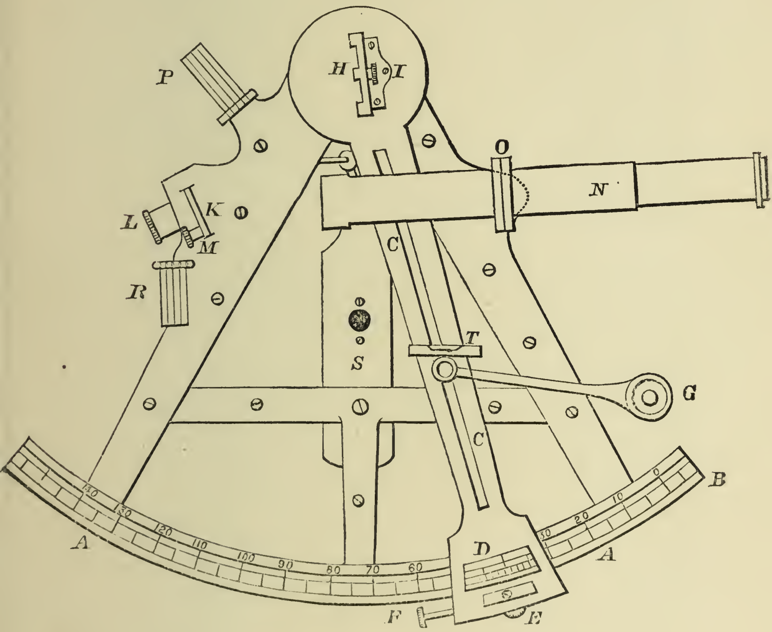
Now if the upper mirror be attached to an index bar, the upper part of which works on a pivot, while the lower end is free to move over a graduated arc adjusted so that when the upper and lower mirrors are parallel the bar points to zero, it is evident that the angular deviation of a ray of light reflected from the two mirrors will be twice the angular distance through which the index bar has moved from zero (see upper arc in Fig. 2); consequently, if the arc be graduated, as all sextants are, to double the angular distance that it actually measures (as in the lower arc shown in the above figure), its indications will correspond with the true angular deviation of a ray that has undergone double reflection from the mirrors of the instrument.

If a portion of the lower mirror be made transparent, enabling the observer to see one object by *direct* vision at the same time that the *reflected* rays from another object enter his eye, the two images may be made to coincide, and the graduation on the arc, indicated by the index bar, will then show the angular distance between the two objects of observation.

THE SEXTANT.

The frame of the sextant is generally made of brass, and should be so fashioned as to combine strength, rigidity, and lightness.

FIG. 3.



A. Limb or arc.
B. Arc of excess.
C. Index bar or arm.
D. Vernier.
E. Clamping screw.
F. Tangent screw.
G. Reading glass.
H. Index glass.
I. Adjusting screw (covered with brass cap).
K. Horizon glass.
L. Adjusting screw (covered with brass cap).
M. Ditto (ditto)
N. Telescope.
O. Telescope-ring or collar.
P. Index shades.
R. Horizon shades.
S. Handle.
T. Knorre's level.

The *arc* or *limb* (*A*) has a strip of metal, silver or platinum, inlaid and running its entire length; this strip

is carefully graduated, usually to divisions of 10′ (ten minutes) each. The lines should be uniformly fine, distinct, and absolutely equidistant. The graduation runs from 0 (zero) towards the left to about 140°, though in instruments of the usual form not more than about 130° can be read with the vernier. From zero towards the right the graduation is carried some 5°; this portion is called the "arc of excess" (*B*), and is used to ascertain the index error of the instrument; it is also useful in determining small angles of terrestrial objects not far from the observer when instrumental parallax affects the observations. A reading from the arc of excess is usually alluded to as "off the arc," in contradistinction to a reading on the arc proper.

The *index bar* (*C*) is a flat bar of brass pivotted at one end on the centre of the instrument, the point coinciding with the centre of the graduated arc. The other end of the index carries a *vernier* scale (*D*), kept close to the graduated arc by means of a strip of brass, beneath the arc, acted upon by a *clamping screw* (*E*), which regulates the pressure required to fix the index at any desired part of the arc. In connection with the clamping screw there is, at right angles to it, a *tangent screw* (*F*), by which, when the index has been clamped, a further but limited amount of motion can slowly and with great regularity be imparted to the index. Rising at right angles from the middle of the index is a small stem carrying the arm of the *reading glass* (*G*), which, travelling with the index, and having an independent lateral motion, can be brought over any desired portion of the graduated arc. A small ground-glass shade is attached to the reading glass to tone the light when a reading is being taken.

The *vernier* (*D*), which in costly and highly finished instruments is often of gold, is soldered to the index, and is usually so graduated as to subdivide the divisions of 10′ on the arc (with which it is concentric) into 10″, which is then the least reading or "count" of the

instrument. With a well-graduated sextant of large radius, in a favourable light, it is possible for an experienced observer to estimate the angle to within 5″. An arrow (↑ or ↑) is marked at the initial point of the vernier, and is called its index or zero.

The *index glass* (*H*) is a plane mirror with its two faces ground parallel; it is set in a brass frame attached to the upper part of the index bar, and placed with its face perpendicular to the plane of the instrument and over the centre of motion of the index bar, by the motion of which its angle of inclination to the horizon glass is changed. An adjusting screw (*I*), protected by a brass cap screwed over it, for the purpose of placing the glass perpendicular to the plane of the instrument, is in some sextants placed at the back of the glass; but as the position of the latter is not very liable to alter, this means of adjustment is often omitted, the maker seeing that the glass is in its proper position before the instrument leaves his hands.

The *horizon glass* (*K*) is fixed to the radius of the frame of the sextant perpendicular to its plane and parallel to the index glass when the index points to zero; the half of the glass nearest the face of the instrument is silvered, and forms a mirror, which receives the rays reflected from the index glass, and turns them back to the eye of the observer; the other half of the glass is left transparent, and it is through this that one of the objects of observation (usually the horizon from which the glass takes its name) is viewed by direct vision. The horizon glass is furnished with screws for making two adjustments: 1st, to set it perpendicular to the plane of the sextant; 2nd, to set it parallel to the index glass when the index points to zero.

The position and number of these adjusting screws vary in different instruments, but the first adjustment is generally made by a single cap-protected screw (*L*) at the back of the sextant frame; the second adjustment by

a smaller cap-protected screw (*M*) at the back of the glass itself, or by means of two capstan-headed screws near the glass, which, acting against each other, slightly turn on its axis the frame holding the glass. Occasionally the means of making the second adjustment is omitted, in which case the amount of the error must be ascertained and allowed for when reducing the observations.

The sextant should be furnished with a plain tube without any lenses, a small non-inverting telescope, and one or more inverting telescopes (*N*), each of the latter provided with two pairs of parallel wires placed in its focus, each pair crossing the other at right angles, and defining, in the middle of the telescopic field, a small square, wherein all observations should be made.

Occasionally, but not often, the inverting telescope is provided with a rack and pinion to facilitate the focal adjustment. Two or more shades of coloured glass, to screw on the eyepiece of the telescope, are required.

A *ring* or *collar* (*O*) with a triangular stem slides into a socket in the radius of the frame opposite the horizon glass; this collar bears a second ring, into which the telescope is screwed, and which can be adjusted by two opposing screws, so as to set the axis of the telescope parallel to the plane of the instrument. These rings, and the telescope within them, can be raised from or lowered towards the face of the sextant by means of a large mill-headed screw at the end of the socket; thus bringing the optical centre of the telescope opposite any desired part, silvered or transparent, of the horizon glass, in a line perpendicular to the sextant plane—an arrangement by which the relative brightness of the reflected and direct images is regulated.

A set of three or four shades (*P*) coloured glasses of different degrees of transparency are so hinged on the frame that they can be separately or in combination placed between the index and horizon glasses. A similar set (*R*) is

hinged on the opposite side of the horizon glass, and can be turned up into the line of sight between the observer and the object viewed through the transparent portion of the horizon glass.

The *handle* (*S*) of wood is fixed at the back of the instrument; it should be bored with a hole, brass bound, $\frac{1}{3}$ inch in diameter, by which the sextant can be attached to a stand.

A small brass leg under each end of the arc, together with a brass protection-cap under the centre of the instrument, support the latter when laid, as it should be, horizontally in its case or elsewhere.

The sextant case should be of well-seasoned wood, and so fitted as to hold the inverting telescope, with the tube adjusted to the observer's focus, as well as the sextant with the index clamped to any division on the arc between 5° and 100°; it should have receptacles for the coloured eye-pieces of the telescope, for spare index and horizon glasses, for a camel's hair brush to clean the mirrors, and for the several telescopes and tubes. The interior fittings, blocks, &c., should be fastened with screws, not merely glued. A brass folding countersunk handle, hooks and studs to keep the lid closed, and a really good lock and key, are requisite.

ON CHOOSING AN INSTRUMENT.

The following are the approximate prices charged by London makers for good reliable instruments divided on silver :—

3 inch radius	divided to	20″	. .	4 guineas to	7 guineas.
4 „	„	20″	. .	5 „	8 „
5 „	„	10″	. .	7 „	9 „
6 „	„	10″	. .	8 „	10 „
8 „	„	10″	. .	9 „	10 „

Superior 8-inch sextants, divided on platinum to 10″, with gold vernier, extra-power telescopes, and with Kew

Observatory certificates, cost from 14 guineas to 18 guineas.

In purchasing a sextant intended for taking with considerable accuracy observations on land, the following points should be attended to:—

1. Choose a sextant that has had its mirrors and shades verified at the Kew Observatory, and see that they bear the mark of that establishment. With a high-priced instrument it is well worth while obtaining the Kew certificate of examination of the finished instrument, which should certify that it is approved for the determination of lunar distances, latitudes, local times, and azimuths, within a maximum error of thirty seconds of arc.

2. The divisions on the arc should be uniformly fine, distinct, and equidistant. Examine the graduation with the reading-glass by a flood of white light, without glare, coming from the sky facing you, and see that when the index is clamped at any part of the arc you have no difficulty, while keeping the eye close to the reading-glass, and *the line of sight perpendicular to the face of the instrument*, in discerning which division, if any, of the vernier coincides with a division of the arc. No more than one division at the same part of the vernier must appear exactly to coincide: if this condition be not fulfilled, the graduation is faulty. If, however, one vernier division appears to fall just a shade to the left of an arc division, while the next higher vernier division seems an equal amount to the right of its corresponding arc division, neither actually corresponding, well and good; for you can then estimate the angle to within two or three seconds, or to less than half the least count of the instrument. Be very careful in making this examination that the optical axis of the reading-glass be brought successively exactly over each division as it is being inspected. It is sometimes advisable, when examining a division, to glance, without moving the head or reading-glass, at the two

adjacent vernier divisions; if they deviate from coincidence with their corresponding arc divisions to an equal extent, the middle division is probably truly coincident with its own arc division.

3. The equality of the distances between the arc divisions should be tested by placing the vernier successively at different parts of the arc and observing whether, when the ↑ is made to coincide with an arc graduation, the division marked 10 also in every instance coincides with an arc division. If it does not do so, the arc divisions are not equidistant.

It is well to note that some verniers are graduated on a small "arc of excess" to the right of the ↑, and also the graduation is sometimes carried to the left a few divisions beyond the highest numbered division, which is usually the tenth.

In reading the instrument be careful not to confound the right-hand division with the vernier index, or zero (↑), when it has an arc of excess.

4. The *tangent screw* should be of sufficient length to afford plenty of run, and must be accurately fitted, to obviate all lost or dead motion (revolution of the tangent screw without corresponding motion of the clamped index arm), though in course of time this defect must, through wear, be expected to develop. Some sextants are fitted with a spiral spring, against which the tangent screw acts, and thus lost motion is avoided.

5. Examine the truth of the surface of the index glass by setting the index near 120°, and observing with the telescope the reflected image of a star of the first magnitude; distinctness and uniformity of outline will be shown by a perfect glass. Absolute truth in the *horizon glass*, though desirable, is not as necessary as in the index glass.

6. Those parts of the sextant near the line of sight, at least, should be browned: whether the whole instrument should be thus treated is a matter of opinion.

Probably, with a bright polished frame, when the sextant has to be exposed to the sun, the errors arising from distortion of the frame as its various parts expand by the absorption of heat, will be less than in a sextant that is entirely browned, and which of course has greater powers of absorption. The error due to the expansion of the sextant under the influence of heat is far from insensible in refined observations.

7. See that the two faces of each shade are parallel, by measuring several times the sun's horizontal diameter, using the shade on the eye-piece of the telescope only; then, removing this shade, turn up an index and horizon shade, and make an equal number of similar measurements: any discrepancy between the two values found by taking the mean of each set of observations exposes an error in the last-named shades. The shade on the telescope eye-piece will not, even if it be imperfect, cause any error.

8. Eccentricity (*i.e.*, non-coincidence of the centre of the graduated arc and the centre about which the index arm revolves) causing errors to the extent of 20″ or 30″, must be expected even in the most carefully made sextants manufactured by the best makers. It is therefore *most important* that the observer should be furnished with a table of corrections to be applied to every tenth or fifteenth degree throughout the arc. As the errors of eccentricity are not easily determined, the purchaser of an instrument should endeavour to obtain from the maker such table of corrections. The eccentricity is found by observing, with the sextant, angles whose true value has previously been determined with a standard theodolite, or by some other means, and comparing the results.

ADJUSTMENTS OF THE SEXTANT.

1. *To set the index glass perpendicular to the plane of the sextant.*—Place the index at about 60°, turn the arc away from you, and hold the instrument nearly horizontal; look obliquely into the index glass, and observe if the arc seen directly and its reflection appear in the same plane; if not, slightly turn the screw *I* till the arc and its reflection seem to form one continuous arc. This adjustment is but seldom required.

2. *To set the horizon glass perpendicular to the plane of the sextant.*—Clamp the index at zero, and observe with the inverting telescope a star—one of the second or third magnitude in preference to a brighter one. Turn the tangent screw, and observe whether the direct and reflected images coincide when passing each other; if they do not, turn the screw *L* till coincidence be obtained. The sun, moon, or indeed any well-defined terrestrial object, can be used in making this adjustment; but a star is to be preferred.

3. *To set the horizon glass parallel to the index glass when the index is at zero.*—This adjustment is not absolutely necessary, as the error arising from an inclination of the two glasses is easily determined, and can be allowed for. Perfect adjustment is by no means easily made, but it is well to have the error (called "index error") conveniently small; so an approximate adjustment should be made in the following manner:—

Fix the index exactly at zero, and observe a second or third magnitude star through the inverting telescope. If the reflected and direct images be not coincident, turn the small screw *M* at the back of the horizon glass till the two images appear as one. Here, as in the case of the second adjustment, the sun, moon, or a well-defined distant terrestrial object, may be employed when there happens to be no star suitable for observation.

4. *To place two of the cross wires of the telescope parallel to the plane of the instrument.*—Observe a star or any point through the telescope, and by placing the index three or four degrees on either side of zero, separate the reflected and direct images so that one of them shall be near the top and the other near the bottom of the telescopic field; bring the edge of one of the vertical wires into contact with the image near the top of the field, and revolve the telescope tube in its cell till the same edge of the same wire is in contact with the lower image. The wire will then be parallel to the plane of the instrument, and the tube and cell may be marked so that the former can on future occasions be at once placed in its proper position.

5. *To set the axis of the telescope parallel to the plane of the sextant.*—Turn the sliding tube of the inverting telescope till two of the wires are parallel to the plane of the sextant, select two celestial objects—as the moon's limb and a star, the moon's limb and the sun's limb, or two stars—whose angular distance is between 100° and 120°; bring the reflected image of one and the direct image of the other into contact at that part of the field of the telescope which is close to the wire nearest the sextant plane; clamp the index, and by slightly changing the position of the instrument bring the images close to the wire farthest from the plane of the sextant. If the contact still remains perfect, no adjustment is required; but if, on the other hand, the two objects appear to separate, slacken the screw in the telescope ring farthest from the sextant frame, and tighten the one on the opposite side; if the images overlap, reverse the operation. If this adjustment be not perfect, the observed angles are always too large.

TO ASCERTAIN THE INDEX ERROR.

1. *By the sun.*—Clamp the index at about 30′, and, holding the sextant horizontally, observe the sun through the inverting telescope, on which the coloured glass eye-piece must be screwed. Turning the tangent screw, make the near limbs of the images slightly overlap if they do not already do so; then, reversing the motion of the tangent screw, make the contact of the limbs as perfect as possible. The point of contact must be in the middle of the square defined by the cross-wires of the telescope. Keep the eye close to the eye-piece and don't change its relative position during the observation. Read (see page 17) the vernier, and note down its indications, giving them the − sign. Next clamp the index at about 30′ on the arc of excess (to the right of zero, 0°), and observe as before; read (see page 17) and note down, giving this observation the + sign.

Then half the arithmetical difference of these two readings is the "index error," and the "index correction" should be applied according to the sign (+ or −) of the greater reading.

Let the readings be as follows:—

			′	″
1st reading, on the arc		−	32	30
2nd ,, off ,,	(on "arc of excess")	+	31	40
		2)	0	50
	Index correction	−		25

Again:

		′	″
1st reading, on the arc	−	31	50
2nd ,, off ,, ...	+	32	10
	2)	0	20
Index correction	+		10

In the first example, the correction 25″ must be sub-

tracted from all observations, to free them from index error; in the second example, 10″ must be added.

If the arithmetical sum of the two readings be divided by 4, the quotient gives the observed semi-diameter of the sun, which, if the observations have been well made, will be found to agree within one or two seconds with the sun's semi-diameter as given in the almanac.

To obtain precision, a number of observations, both on and off the arc, should be taken, and the mean of each set used instead of a single pair of readings.

Example.—On June 14, 1885, the following measures were taken:—

On the Arc.	Off the Arc.	
′ ″	′ ″	
− 32 10	+ 30 50	
32 10	30 55	
32 15	31 5	′ ″
32 5	31 0	32 8·57
32 0	31 0	31 0·71
32 15	31 5	
32 5	31 10	4)63 9·28
7)225 0	7)217 5	15 47·32 Obsd. semi-diam. of sun.
− 32 8·57 Means	+ 31 0·71	′ ″ 15 46·70 S D. of sun ℞ Almanac.
+ 31 0·71		
2) 1 7·86		
− 33·9 Index correction.		

N.B.—In determining the index correction by the above method, be careful not to expose the sextant to the rays of the sun more than is absolutely necessary, as the error alters perceptibly when the temperature of the instrument changes to any great extent, and should altitudes of the stars, planets, or moon have to be observed in the cooler temperature of night, the correction obtained when the sextant frame was expanded through exposure to a hot sun will not be applicable, and it will therefore be necessary to re-determine the index error.

2. *By a star.*—Clamp the index at 0°, and observe with the inverting telescope a second or third magnitude

star; bring the direct and reflected images into coincidence by means of the tangent screw, and read the vernier. This reading is the index error, and the correction is numerically equal to it: positive, when the reading is off the arc; minus, when on the arc.

The observation for the correction by this method is not so easy to make with accuracy as the solar one, but the method will often be found useful.

The index correction can also be approximately determined by observing in a similar manner a well-defined *distant* terrestrial object—the sea horizon, for instance.

CARE OF THE INSTRUMENT.

In removing the sextant from its box, lift it by the frame or the handle; never by the index bar.

When not in use the index should be kept lightly clamped to the arc.

Never force the tangent screw when it is at the end of its run, or when the index bar is near either extremity of the arc, where it is liable to come in contact with the frame holding the horizon glass or the stem of the telescope collar.

Clean the index and horizon glasses as seldom as possible, and then only use the very softest material for the purpose; the less these glasses are touched the better, as they are easily put out of adjustment. To remove dust, &c., brush them carefully with a camel's-hair brush kept for the purpose.

After cleaning the glasses, and before observing an angle, test the adjustments, and verify the index correction.

The telescope lenses, the shades, and reading glass may be cleaned with a piece of chamois leather; this material should also be used for polishing the arc and frame. Be careful that not the minutest particle of mercury from the artificial horizon be allowed to touch

the graduated arc or vernier, for a stain will result from such contact. The sextant must never be touched by a leather which has been used to strain the horizon mercury.

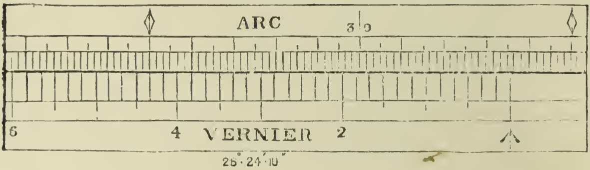
Guard the instrument from all concussion, and never trust it in the hands of a person unacquainted with the delicacy of astronomical instruments used for angular measurement.

Keep its box locked when the sextant is in it.

READING THE ARC.

The arc is usually graduated to divisions of ten minutes (10′) each, which are subdivided by the vernier to ten seconds (10″).

Fig. 4.



On the arc itself the main graduations indicate degrees (°), every fifth being cut somewhat longer than its fellows, and marked †, while each tenth graduation is engraved with its proper number.

Every degree is subdivided by five shorter lines into six divisions of ten minutes (10′) each; the middle line of the five being rather longer than the others indicates the half-degree, or 30′.

On the vernier the larger graduations represent minutes (′), every alternate one being engraved with a number showing its value.

Each minute is subdivided, by five shorter graduations, into six divisions of ten seconds (10″), which is the least count of the instrument.

To read off an observation, note where the zero (↑ or ↑) of the vernier rests, and observe how many whole degrees and subdivisions (of ten minutes each) lie between it and the zero of the arc. Suppose 26° and two subdivisions are thus found to be to the right of the vernier ↑, then the reading is 26° 20′. Probably the vernier ↑ will be found somewhat to the left of its corresponding arc graduation, in which case run the eye along the vernier lines until one of them is found to coincide with an arc graduation. The angular value of this line from vernier ↑ will be apparent, as the figures engraved on the vernier indicate the number of minutes, and the smaller lines the number of subdivisions of ten seconds (10″) each. This value must be added to the reading already taken from the arc. Suppose the coincident vernier line shows that 4 minutes and one subdivision lie between it and ↑, then 4′ 10″ is the vernier reading to be added.

The whole reading will be—

		°	′	″
On the arc		26	20	0
On the vernier . . .	+	0	4	10
Instrumental reading . .		26	24	10

To read an angle on the "arc of excess" (which seldom exceeds 5°), read from left to right the number of degrees and minutes on the arc between its zero and the vernier ↑, observe the vernier as before directed, subtract its reading from 10′, and add the difference so found to the arc reading.

Suppose the arc reading to be 1° 30′ and the vernier reading 8′ 20″, then—

		°	′	″
	Arc . .	1	30	0
10′ − 8′ 20″ = 1′ 40″	Vernier . .	0	1	40
	Instrumental angle . .	1	31	40

Or, if preferred, the vernier can, like the arc, be read from

left to right, commencing at 10′ instead of at ↑, and counting the minutes and seconds between the former and the coincident line; add these direct to the arc reading.

In reading off an angle with the reading glass the importance of keeping the line of sight absolutely perpendicular to the plane of the arc cannot be overestimated.

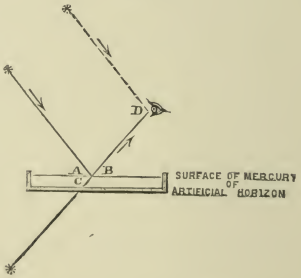
If this precaution be neglected, the error arising from parallax will be considerable.

THE ARTIFICIAL HORIZON.

This is a highly reflecting plane surface placed in a truly horizontal position. Quicksilver (mercury) or any viscous fluid protected from the wind, or any force which would impart a tremor to its surface, can be used.

The angle between an object and its reflection from the artificial horizon is double the altitude of the object from the observer's *sensible* horizon, therefore if the former be measured with the sextant, or other instrument, and the reading halved, the observer obtains the observed altitude of the object, which is what he requires for his calculations.

Fig. 5.



Suppose a ray from a star be reflected from the mercury of an artificial horizon to the eye; the direct rays from the star to the eye will fall in the direction of

the dotted line, the small space between the eye and the mercury is insignificant in comparison with the immense distance of a star; therefore, the rays falling on the eye and on the mercury respectively may be considered parallel. The reflected image of the star will appear to the observer in a continuation, below the mercury, of the straight line between the eye and the artificial horizon.

Now the angles of incidence and reflection, A and B, are equal, the opposite angles B and C are equal, $\therefore A+C=2B=2A$; and since the rays from the star to the mercury and from the star to the eye are practically parallel, the angle $D=A+C=2A$.

If, in place of a star, a terrestrial object only a few yards distant be observed, a sensible error will be introduced through the eye being away from, instead of at, the surface of the mercury.

The most reliable form of the artificial horizon is a shallow trough containing mercury, protected from disturbing influences of air currents by a cover in which are framed two carefully ground plates of glass, whose planes are approximately at right angles to each other, and at an angle of about 45° to the surface of the mercury when the cover is placed in position for use. Each glass must have its two faces parallel, otherwise an error in the observations will be introduced. The glasses should be carefully tested before the instrument be purchased. This instrument packs into a small box containing also an iron bottle in which the mercury is kept when not in use.

Captain George's horizon consists of a circular iron trough containing mercury, on which a disc of glass, with parallel faces, floats. It is portable and convenient to use, but unless care be exercised when placing the glass on the mercury, and unless the latter be fairly pure and clean, rather large errors, quite unsuspected by the observer, may be introduced. I have known instances where errors of 3′ 45″ and 7′ 33″ occurred in latitudes

deduced from observations taken with a carelessly floated glass. Before floating the glass a piece of thin paper should be placed on the mercury, the glass should then be put on it and lightly pressed down while the paper is carefully withdrawn.

These horizons are fitted with a glass lid, which screws on to the top of the trough, and I have found that in windy weather, by discarding the floating glass and using the glazed lid only, I obtained results leaving nothing to be desired as regards accuracy, though care had sometimes to be taken not to mistake an image reflected from the glass lid for the one reflected from the mercury.

In Captain George's horizon the mercury when not in use is carried in a circular cistern communicating with the trough by an orifice fitted with a small tap, an arrangement which obviates the danger of spilling any of the mercury when setting up the instrument for observation or when packing it up.

A third form of horizon consists of a plane mirror, which is adjusted horizontally by means of a spirit level. Where accuracy is desired such an instrument cannot be recommended although it is exceedingly portable and comparatively cheap and light.

It is desirable that the mercury used in every form of the artificial horizon should be clean, and offer a good reflecting surface. It should, when necessary, be strained through thin chamois leather or two or three folds of clean silk.

It will be found that the border of the mercury in the trough is convex owing to capillary attraction : this portion must not be used for observation, or error will result. Sometimes it happens that the reflection of a celestial body under observation will, owing to its apparent motion, travel unnoticed to this part of the artificial horizon, in which case the definition and distinctness of the image will become impaired and the direction of the reflection changed ; but the inexperienced observer

is liable to ascribe the alteration in appearance to other causes, such as dew on the artificial horizon glasses, imperfection in the focussing of the telescope, &c., and thus either allows his observation to be vitiated by an unsuspected source of error, or will perhaps only discover that he has been using the wrong part of his horizon when it is too late to obtain the observation he desires.

It is one of the advantages of the floating-glass horizon that this defect of the uncovered mercury is remedied, the whole surface of the horizon being available for observation.

When the state of the weather permits the reflections should be observed from uncovered mercury, as under these conditions alone are observations thoroughly reliable; such opportunities, however, occur but seldom.

When the altitude of an object is small the area of the artificial horizon is much curtailed to the observer; so an instrument of not less than $3\frac{1}{2}$ inches in diameter should be selected.

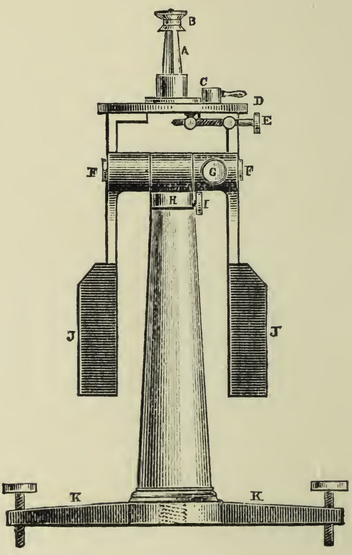
THE SEXTANT STAND.

The accuracy of sextant observations is greatly increased by means of a stand so constructed that the sextant, when attached to it, can be fixed in nearly any required plane. By using a stand the observer has one and often both hands free to manipulate watch, notebook, pencil, lamp, &c., while actually observing an altitude or lunar distance; an advantage of some importance when he has no reliable assistant to note the time for him. If an observer does not desire very great accuracy, and if he is satisfied with results giving the latitude within a quarter of a mile, and the longitude, by lunar distances, within ten miles (on the equator) of the truth, he can manage without a stand.

The disadvantages of the sextant-stand are its weight, bulk, and expense.

Although, perhaps, it is an adjunct more convenient than necessary (except in observations of refinement), I would advise any traveller who wishes to obtain reliable results, and who may have, without an assistant, to take observations, to invest in a stand of simple form. He will find the additional ease and comfort its use affords

Fig. 6.



Porter's Improved Sextant Stand.

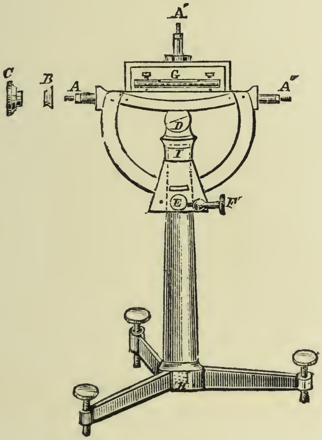
A. Square pin fitting through handle of Sextant.
B. Screw-nut and washer to clamp Sextant handle to pin A.
C. Clamp screw to revolving pin A.
D. Circular plate and centre holding square pin A.
E. Tangent screw for slow motion to pin A.
FF. Horizontal centre.
G. Clamp-screw on centre FF.
H. Vertical centre of pillar.
I. Clamp-screw to horizontal motion.
JJ. Counterpoise weight.
KK. Tripod foot and screws.

conducive to accuracy, while all fatigue to the hand and arm, with its accompanying tremor of the hand-supported sextant, is avoided.

Stands are constructed of brass, and are of various forms. In one description a vertical pillar is supported by a tripod, each foot bearing a large-headed adjusting screw, the point of which rests on the table, stool, or other flat surface on which the stand may be placed.

A cap, capable of being revolved round a vertical axis, is fitted on the head of the pillar, to which it can be clamped by means of a collar with a tightening screw. This cap carries, on a horizontal axis, allowing a vertical motion, a frame at one end of which is a pin, to which

FIG. 7.



the sextant can be fastened by means of a screw-nut and washer; while at the other end a pair of counterpoise weights are placed. In Porter's improved stand the pin passing through the sextant handle is square, and revolves on the circular plate of the frame, to which it can be clamped, and a slow motion imparted to it by a tangent screw.

Some years ago, Mr. Henry Porter, 181 Strand, London, made me a stand on the principle shown in figure 7.

A, *A′*, and *A″*, are the arms, on any one of which the

sextant can be fastened by the washer *B* and screw-nut *C*; they move vertically round the centre *D*, and can be fixed by the clamping-screw *E*, which has a tangent screw *F* attached.

The whole turns horizontally on the head of the pillar near *I*, to which it may be clamped by a screw, not shown in the figure, and which tightens a collar encircling the pillar.

G is a spirit-level, for which, however, there is but little need. The pillar unscrews at *H* for convenience in packing. In using a stand of this description, which has no counterpoise, care must be taken to support by the hand, whenever the clamping screw *E* be loosened, the sextant; otherwise it is liable to fall against the pillar and become seriously damaged. The price of such a stand for an 8-inch sextant is from £6 16*s.* 6*d.* to £8 8*s.* The weight with case, 19 lbs.; without case, 12 lbs.

KNORRE'S LEVEL.

If the observer be provided with a stand, he will find a small attachment to the sextant, known as *Knorre's level,* very convenient when observing the altitudes of stars with an artificial horizon, as it avoids the sometimes difficult operation of "bringing down" a star to coincide with its image reflected from the mercury.

It consists of a small spirit-level (see *T*, Fig. 3, page 3) attached to the index-arm, and so adjusted that the bubble will play when the two images of a star reflected from the sextant glasses and from the artificial horizon respectively coincide in the middle of the field of the telescope.

In using it the reflection of the star from the artificial horizon is viewed through the sextant telescope, and the index moved till the bubble plays, when it is clamped. The two images of the star will then be seen nearly if

not actually coinciding in the field of the telescope, if the sextant be properly held with its plane in a vertical position.

The adjustment of Knorre's level is made by bringing the two images of an object into coincidence in the middle of the field of the telescope, and then fixing the level with the bubble in the middle of its run.

OBSERVING WITH THE SEXTANT.

In proceeding to take an observation the following points should receive attention.

The tangent screw should be adjusted near the middle of its run.

The shades, except those in use, should be turned out of the path of the rays proceeding from the object to the observer's eye.

When an artificial horizon is used it should be placed in a sheltered position on the ground, or on a firm support, such as an *isolated* table, box, or stool. The sextant stand, if one be used, should have a separate support, for if placed on that which bears the artificial horizon, the manipulation of the sextant, even the act of turning the tangent-screw, is liable to impart a tremor to the mercury which will often, in the case of a meridian altitude, cause the loss of the observation.

The adjustment of the focus of the telescope must be perfect, and two of the threads or wires placed parallel to the plane of the sextant.

The two images should be made of equal intensity or brightness by means of suitable shades, by regulating the distance of the telescope from the plane of the instrument, or by so holding the sextant as to observe the fainter object by direct vision, and thus reduce the intensity of the brighter object by the loss of light due to its double reflection from the mirrors.

When observing the sun with the artificial horizon a suitable shade on the eyepiece of the telescope only should be used, thus avoiding all chance of shade error.

Care should be taken not to have the images of sun or moon too bright; the fainter the better, as long as they are perfectly distinct.

On a dark night, in observing stars in the artificial horizon, *the mercury should be slightly illumed by artificial light,* otherwise the cross wires in the telescope will not be visible, and consequently the observer will not be able to discern whether or not he is making the contact of the images in the proper manner, in the middle of the field of the telescope.

On the other hand, when the moon is very bright the artificial horizon should be protected from its direct rays as much as possible in stellar observations, otherwise the light from a faint star may be partially quenched in the lustre of the mercury's surface.

The observer should assume as stable and comfortable a position as possible, and when working without a stand should contrive to rest each arm upon some firm support. In observing small altitudes with an artificial horizon, by lying prostrate, resting both elbows on the ground, considerable steadiness is obtainable. In taking altitudes of objects near the zenith, sitting on a low box or stool, and resting the forearms on the knees, will often be found convenient; while in taking lunar distances it will be necessary to assume whatever position affords, without constraint, the best means of gaining support for the arms and steadiness for the head.

In sextant observations an easy position is just as essential as a constrained one is fatal to accuracy.

Ample time for preparation should always be afforded, especially when a meridian altitude is to be observed; one often lacks confidence in an observation hurriedly taken, and is apt to assign to it less weight than it may happen to be entitled to. Observe and always keep in

mind which way the tangent screw must be turned to "bring down" a rising object, and *vice versâ*.

The indications of the barometer and thermometer should invariably be noted, for the purpose of ascertaining the corrections which must be applied to the refraction extracted from the Tables. (See Tables I. II. and III.)

To observe an altitude of a celestial object when using an artificial horizon, place yourself in a convenient position for seeing the reflected image of the object (which for convenience we will suppose to be the sun) in the middle of the surface of the mercury.

Hold the index, unclamped, at zero, and protecting the eye with the necessary shades, look directly at the sun through the collar used for carrying the telescope and through the transparent part of the horizon glass, slowly move the index forward, and as the two images separate gradually lower the sextant, keeping the reflected image in view till you have brought it down to the artificial horizon and made its limb (either upper or lower) touch the opposite limb of the image seen in the mercury; clamp the index, promptly screw into its collar the inverting telescope, and observe in the *middlè* of its field the two images; make contact perfect by means of the tangent screw, observing that when slightly moving the sextant round the line of sight the limbs in passing just graze each other. If the shade on the telescope eye-piece is used, of course those attached to the frame must be turned out of the way after the sun's image has been brought down.

A star's altitude is observed in a similar manner, but no shades are used: the images in a good telescope appear as mere bright points, so there are no limbs to observe. The two images are made to coincide and appear as one.

The eye should be kept in the middle of, and close to, the telescopic eye-piece.

If a stand be used, the sextant may be fastened to it after the sun or other body has been brought down and

the index clamped. When a Knorre's level is provided the sextant may be attached to its stand from the beginning, and the observation made in the method described at page 24.

The use of the inverting telescope is at first apt to perplex an inexperienced observer, leaving him uncertain not only which limb of the sun, but also which of the reflected images he is actually viewing. A light tap on the artificial horizon, to shake the mercury and temporarily destroy the truth of the reflection, will put him right on the latter point, and by bearing in mind that everything he sees is inverted—that what appears to be the upper limb of an image is really its lower limb, and that that which is apparently rising is in reality falling—he will soon find no great difficulty in identifying the images or their limbs.

If, when observing a star in the artificial horizon, the sextant be slightly moved round the axis of the telescope, the image reflected from the mercury will remain stationary, while the other reflected from the sextant mirrors will appear to travel across the field of view. This method of identification is of constant use in observing the meridian altitude of a star.

In observing the altitude of an apparently fast-moving celestial body it will generally be found advantageous, instead of making contact of the images with the tangent screw, to fix the index so that a small space is left between the approaching points of contact, and wait until the apparent motion of the heavenly body (the diurnal motion of the earth) causes the contact to be made, when the time should be instantly noted. By this means contact is made with greater regularity than could possibly be obtained by manipulation of the tangent screw, and the observer has his left hand free either to steady the sextant, if working without a stand, or to hold his watch in a favourable position for noting the time immediately the contact takes place.

To observe an altitude from the apparent (sea) horizon, the image of the object should be brought down in the manner already described till it touches the horizon line at the point vertically under it. After screwing in the telescope observe the object in the middle of the field, and vibrate the sextant about the line of sight, making the image describe an arc.

With the tangent screw make perfect the contact when the image is at the lowest point of the arc, if a non-inverting telescope be used. With an inverting telescope the contact must be made at the apparently highest point of the arc, which appears with its convex side upwards.

In observing altitudes for determining the time, it is as well to allow the contact to be perfected by the apparent motion of the object.

In ascertaining altitudes or distances of the moon the bright or perfect limb is the one observed. In solar observations either limb may be used, but I have found the upper one gives the most accurate results, the error due to the optical illusion, which causes the sun (in common with all bright objects) to appear larger than it really is, balancing another small error, the source of which is uncertain.

To measure the angular distance between the moon and the sun.—Turn up a suitable index shade, and keeping the index at zero, view the moon through the telescope ring and the horizon glass, holding the sextant with its face upwards or downwards according as the sun appears to the right or left of the moon, and in such a position that the plane of the sextant is, as nearly as can be judged, parallel with a line joining the sun and moon. Move the index slowly forward until the nearest limb of the sun's image be seen in contact with the enlightened limb of the moon, which is always the one nearest the sun; clamp the index, screw in the inverting telescope, and by interposing suitable shades and regulating the dis-

tance of the telescope from the plane of the instrument, make the intensity of the two images as equal as possible. Move the sextant slowly round the line of sight, so that the image of the sun appears to pass by the moon, and make in the middle of the field of the telescope the contact perfect by means of the tangent screw; or, preferably, make it approximately, and allow it to be perfected by the moon's motion, noting the exact time when the limbs appear to touch. If the latter course be pursued, a small space must be left between the limbs if the distance be decreasing, while the images must be made to slightly overlap if the distance be increasing: reference to the Nautical Almanac will show whether the sun and moon are approaching or receding.

To measure the angular distance between the moon and a star or a planet.—Observe the star or planet by direct vision, and follow the above directions, substituting the moon for the sun. The contact must be made between the star and the enlightened or perfect limb of the moon, which may happen to be either the nearest or farthest one from the star. In the small telescope of a sextant a star seldom appears as a well-defined point; so the centre of the star's light must be taken as the point with which the moon's limb must be brought in contact. In like manner, the estimated centre of a planet may be observed in preference to measuring the distance to the planet's limb, and allowing for its semi-diameter in subsequently "clearing the distance."

In observing lunar distances with the aid of a sextant stand, instead of working as above directed, it is sometimes convenient to observe the *brighter* object by direct vision, the disadvantage arising from the loss of light from the fainter object (due to its double reflection from the mirrors) being more than counterbalanced by the fact that the sextant attached to its stand may be adjusted in a more convenient position for the observer. When such is the case the equal intensity of the two images must be

obtained by the judicious employment of the horizon shades, and by reducing the distance between the telescope and the plane of the instrument as much as the regulating screw will admit.

It is a good plan, before commencing an observation, to clamp the index at the approximate lunar distance, ascertained by calculation from the data contained in the Nautical Almanac. With a fair knowledge of the Greenwich time, the distance can easily be deduced with sufficient accuracy to ensure both objects appearing in the field of view when the index has been set by this means.

When the moon is near its full it is difficult to determine which is the perfect limb. The Almanac should then be consulted, and the local time of full moon computed. The perfect limb will be the upper one if the moon be rising, the lower one if falling, before the time of full moon. When the moon is waning, the opposite limb, being then towards the sun, will be the perfect one.

To observe the angular distance between two terrestrial objects.—Setting the index at zero, look through the telescope collar at the less distinct of the two objects, hold the sextant with its plane parallel to the plane containing the objects and the eye, and with its face upwards, if the more distinct object is to the right—downwards if it is to the left of the object viewed by direct vision; move the index slowly forward till the image of the object reflected from the mirrors appears to coincide with the object seen through the transparent part of the horizon glass, clamp the index, insert the telescope, and make the contact perfect with the tangent screw. The reading, after being corrected for instrumental errors, is the angle subtended by the distance between the two objects, but of course it is not the true horizontal angle which would be required for accurately plotting a survey, unless both objects are on the same level, or unless

one of them is in the same horizontal plane as the observer, and the angular distance between them is exactly 90°.

To obtain the horizontal angle, when the objects are at different altitudes select by means of a plumb-line, or by estimation, a point vertically beneath the higher object, and on the same level as the lower one, and measure the angle subtended by this point and the lower object; or, if the angular distance does not exceed 20°, the horizontal angle can be obtained with sufficient accuracy by selecting some mark situated in the observer's horizontal plane, and about 90° from the object nearest to it, and measuring successively the angles between the two objects and the mark; the difference between the readings will be the true horizontal angle.

In sextant observations the observer's eye is seldom in a continuation of the straight line joining the reflected object and the middle of the index glass, consequently the angles are slightly erroneous on account of *instrumental parallax*, when both objects happen to be only a short distance, say less than a quarter of a mile, from the observer. The instrumental parallax equals the angle at the object reflected from the mirrors, subtended by the small distance between the eye and the middle of the index glass.

If only one of the objects is nearer than a quarter of a mile, parallax can be avoided by viewing the more distant object by reflection, as no error is introduced by observing a near object directly *through* the horizon glass.

In observing near objects, when the angle subtended by them is not too large, instrumental parallax may be eliminated by measuring the angles both *on* and *off* the arc, and taking the mean of the two readings; in this case no correction for index error is necessary, as the latter is eliminated together with the parallax.

To measure an angle on and off the arc, observe one of

the objects directly through the horizon-glass, make the other by reflection coincide with it, and note the reading; then invert the sextant, move the index to the opposite side of zero to that at which it was previously fixed, make the images coincide, and read off a second time.

As the arc of excess is limited to a few degrees, only small angles can be measured by this method.

The parallax may also be nearly eliminated by applying to the instrumental reading of the angle subtended by the objects, a correction involving both index error and parallax. This correction is determined in the same manner as the index error is found when using a star or terrestrial object—viz., by bringing into coincidence its reflected image, and that object which, when subsequently measuring the angular distance of the two objects, is viewed directly through the horizon-glass. If the object were at a considerable distance, the reading would give the index error only; but when the object is so near that the sextant parallax is appreciable, it gives the index error and the parallax combined.

Astronomical observations with the sextant are made chiefly for the purpose of determining—

Latitude:

- 1st, By meridian altitudes.
- 2nd, By circum-meridian altitudes; *i.e.*, several altitudes near and on each side of the meridian, the time being given.
- 3rd, By a single altitude near the meridian, the time being given.
- 4th, By an altitude of the Pole Star at any known time, and by other methods less frequently employed.

Time:

- 1st, By a single altitude.
- 2nd, By equal altitudes.

Longitude:

1st, By a comparison of local mean time deduced from observation, with Greenwich mean time found by chronometer.
2nd, By lunar distances.
3rd, By the moon's altitude.

The variation of the compass:

1st, By a single altitude.
2nd, By equal altitudes.

The direction of the meridian line:

By the angular distance of the sun from a terrestrial object.

LATITUDE.

(*See Definition.*

By meridian altitude of the sun.—Commence observing the altitude of either limb (I prefer the upper for reasons already stated) about ten minutes before the estimated time of *apparent* noon, and as the sun rises maintain perfect contact till the time of culmination, by the tangent screw, never reversing its motion. Directly the sun begins to "dip" the altitude should be read off.

Owing to the sun's change of declination, the observed altitude will generally be slightly in excess of a strict meridian altitude, but the error, even at the equinoxes, when it is greatest, is practically insensible in sextant observations.

Example 1.—On September 18, 1885, in estimated longitude 32° 30′ E., with an artificial horizon, the meridian altitude of the sun's *lower* limb (symbol $\underline{\odot}$) was observed to be 113° 26′ 30″. Sun south of observer. Barom. 30 in. therm. 75°.

	H.	M.	S.
Apparent noon, Sep. 18 . .	0	0	0
Long. 32° 30′ E. in time (Table X.)	2	10	0
Greenwich app. time, Sep. 17 .	21	50	0

Sun's declination (decreasing).

	°	′	″	
At Greenwich appt. noon, Sep. 18	1	42	29·4	N.
Correctn. for var. 2h. 10m. before noon		2	6·2	+
At time of observation . .	1	44	36	N.

Var. of decl. in 1 hour, 58″·19
× by 2·17 time from noon in hours and decimals
= 126″·2 = 2′ 6″·2

°	′	″		
113	26	30		Double "instrumental" alt. of $\underline{\odot}$
		22	−	Correction for index error.
113	26	8		
0	0	0		Correction for sextant eccentricity.
2)113	26	8		
56	43	4		"Observed" altitude.
		36	−	Corrected refraction (Table .).
56	42	28		
		5	+	Parallax (Table V.).
56	42	33		True altitude of $\underline{\odot}$
	15	58	+	⊙ semi-diameter per Almanac.
56	58	31		True altitude of ⊙ centre.
90				
33	1	29		Zenith distance N.
1	44	36	+	Corrected declination N.
34	46	5		North latitude.

Example 2.—On April 11, 1885, at the same station as that of the observation in Example 1, the meridian altitude of the sun's upper limb ($\overline{\odot}$) was observed in artificial horizon, and found to be 127° 56′ 0″. Sun south of observer. Barom. 30 in.; therm. 60°.

	H.	M.	S.
Apparent noon, April 11 . .	0	0	0
Long. 32° 30′ E. in time Table X.)	2	10	0
Greenwich app. time, April 10 .	21	50	0

Sun's declination N. (increasing).

	° ′ ″
At Greenwich appt. noon, April 11	8 28 54·2 N.
Correction for 2h. 10m. before noon	1 59·2
At time of observation . .	8 26 55 N.

Var. of decl. in 1 hour, 54″·93
× 2·17 hours = 19″·2 = 1′ 59″·2

°	′	″		
127	56	0		Instrumental 2 alt. ⊙
		37	–	Correction for index error.
127	55	23		
0	0	0		Correction for eccentricity.
2)127	55	23		
63	57	41		"Observed" altitude.
		28	–	Corrected refraction (Table I.).
63	57	13		
		4	+	Parallax Table V.).
63	57	17		True altitude ⊙
	15	59	–	⊙ semi-diameter
63	41	18		True altitude ⊙ centre.
90				
26	18	42		Zenith distance N.
8	26	55		orrected declination N.
34	45	37		North latitude.

When the place of observation is in west longitude, the time then being after Greenwich noon, the variation of the declination must be subtracted from a decreasing declination and added to an increasing one.

When the sun is north of the observer the zenith distance is south.

In deducing the latitude, in all cases the zenith dis-

tance and the declination must, if they are both north or both south, be added together; if one be north and the other south, the difference will be the latitude of the same name as the greater.

When the atmosphere does not vary much from its mean state—*i.e.*, when the barometer stands at 30 inches and the thermometer at 50°—the refraction may be applied uncorrected to altitudes over 20° without serious error resulting.

It will be seen in the above examples that, although the observations were taken at the same station, there is a difference of 28″ between the deduced latitudes. The mean of the two values gives 34° 45′ 51″, which is very nearly the true latitude found by more accurate methods. In the examples, opposite limbs of the sun were observed, and the discrepancy of 14″ between the mean value and either latitude is in a great measure due to the optical enlargement of the sun's disc, an illusion inseparable from observations of a luminous object made with telescopes of comparatively feeble power, such as those furnished with sextants. By taking the mean of values derived from observations of the sun's upper and lower limb respectively, this error is eliminated.

The amount of error for any particular telescope (when the same shade is invariably used) is easily found by comparing the results deduced from a few upper-limb and the same number of lower-limb observations respectively. By applying a corresponding correction to all single solar altitudes, the accuracy of the deductions may be enhanced.

The known error for the telescope with which the observations given in Examples 1 and 2 were made was 14″·9 in the double altitude, or 7″·45 in the resulting latitude. If, then, in Example 1, which gives by the lower limb the value 34° 46′ 5″, the correction of 7″·45 be subtracted (since on account of the "instrumental" altitude being 14″·9 too small, and the "observed" altitude 7″·45 too small, the derived latitude is too great

to a corresponding extent), we have $34^\circ\ 45'\ 57''\cdot4$ agreeing with the mean value, $34^\circ\ 45'\ 51''$ within $6''\cdot4$; representing about 216 yards of the earth's surface. This discrepancy is by no means great when we consider the probability of the introduction of errors arising from distortion of the instrument exposed to the heat of a meridian sun, imperfect observation, and perhaps the uncertain refraction due to an abnormal state of the atmosphere.

When the meridian altitude is observed from the sea horizon, the instrumental reading, corrected for index error and eccentricity, gives at once the observed altitude, to which a correction for the dip of the horizon (depending on the height of the observer's eye above the level of the sea) must be applied in addition to the corrections shown in the foregoing examples. Owing to the influence of terrestrial refraction on the sea horizon, the fluctuations in the dip caused by the motion of a ship, and to other causes, extreme precision cannot be attained in observations made at sea. In nautical practice, corrections which amount to only a few seconds of arc are generally neglected, while Tables combining corrections in a convenient but not rigidly accurate form are often used.

By meridian altitude of a star.—This method has many advantages not possessed by the solar one. For instance, the index error of the sextant in night observations is not so liable to change as it is in those taken in the day; the declination of a star being practically constant for intervals of time not exceeding one or two days, no correction need be applied to it, consequently a knowledge of the Greenwich time of observation is not required; and as in the course of a night several bright stars at convenient altitudes cross the meridian, the observer is not restricted to a single daily observation, as he is when he depends on the sun, which, further, in low altitudes is for a great portion of the year beyond the range of the sextant and artificial horizon.

Again, as some stars pass north and others south of the zenith, by observing one (or a series) of each on different sides of the zenith the mean of the deduced latitudes will to a great extent be freed from *constant* errors of the instrument and observer, and from errors of refraction.

In the computation of latitude from a stellar observation there is no correction for semi-diameter or parallax, the immense distance from the earth of a star rendering the magnitude of the latter insensible, and the radius of the earth insignificant.

Example 3.—On Oct. 5, 1885, with an artificial horizon, the barom. at 29 inches, the therm. at 80°, the following meridian altitudes were observed :—

β Ceti south of zenith ; α Ursæ Minoris (*Polaris*) north of zenith.

° ′ ″		° ′ ″
73 17 40	. . Instrumental 2 alts. .	72 11 10
26 −	. . Index correction . .	26 −
73 17 14		72 10 44
15 −	. . Correction for eccentricity	15
2) 73 16 59		2) 72 10 29
36 38 29·5	. . Observed altitude . .	36 5 14·5
1 14	. . Refraction (Table I.) .	1 15
36 37 15·5	. . True altitude . .	36 3 59·5
90		90
53 22 44·5 N.	. Zenith distance . .	53 56 0·5 S.
18 36 44·5 S.	. Declination . . .	88 41 53·1 N.
34 46 0 N.	. Latitude . . .	34 45 52·6 N.

	° ′ ″
Latitude by star north of zenith	34 45 52·6 N.
,, ,, south ,,	34 46 0 N.
	2) 69 31 52·6 N.
Mean latitude .	34 45 56·3 N.

Here either latitude differs with the mean value only by 3″·7, a near agreement which, without being conclusive (for both observations might be grossly inaccurate,

and yet agree like any other false witnesses), leads us to believe that the observations were satisfactory, and that the instrumental corrections had been well determined.

Let us suppose that the existence of index error and of sextant eccentricity had been unsuspected, and that no corrections for them had been applied; then the North Star would have given the latitude as $34° 46' 13''·1$ and the South Star as $34° 45' 39''·5$, both some $17''$ wide of the truth, yet the mean of the two values would be, as before, $34° 45' 56''·3$. Clearly the advantage of thus balancing observations is very great; in fact, the method is indispensable where precision is aimed at.

Very great accuracy in the results cannot be expected with a single pair of observations, but as "the precision of the mean of a number of observations increases as the square root of their number," by increasing the number of pairs of observations, and taking the mean of the deductions such a close approximation to absolute truth as the capabilities of the instrument permit, may finally be arrived at. It must be borne in mind, however, that the least count of the sextant being generally $10''$, an observer cannot, even under the most favourable circumstances, be sure of estimating the reading to within less than $2''·5$; this amount is reduced by one-half when an artificial horizon without error is used. "We cannot measure what we cannot see;" so $1''·25$ may be considered the ultimate limit of accuracy of values derived from altitudes observed with the sextant, the instrument being of course in perfect adjustment, and the index error and eccentricity accurately known. For the purpose of exhibiting, in cases where due skill and care have been exercised, the variance between different observations and the mean value, I subjoin the result of twenty meridian altitudes of stars, observed to determine the latitude of the station at which were taken all the observations shown in the examples in this treatise. The sextant was of 8-inch radius and was attached to a stand.

	Star North.				Star South.				
	°	′	″		°	′	″		
	34	45	59	N. lat.	34	45	56	N. lat.	
			52				52		
			46				54		
			55				41		
			53				53		
			54				50		
			53				55		
			51				54		1 minute is here given in equivalent terms of seconds for convenience of computation.
			50				49		
			46				60		
10)			519		10)		524		
Means by Star North	34	45	51·9		34	45	52·4		
,, ,, South	34	45	52·4						
2)	69	31	44·3						
Mean of means	34	45	52·1	N. lat.					

which is probably true within at most 2″ of latitude, or 66 yards on the earth's surface.

By circum-meridian altitudes of a star.—This method gives very precise results when the time has been well determined, as it may be by an observation of some star, favourably situated near the prime vertical, *immediately* before or after the circum-meridian observation is taken. The time obtained by an ordinary watch, whose error has been found three or four hours previously, cannot be implicitly relied on, however regular the watch's daily rate may appear to be. Watches (I am not speaking of chronometers) will often gain or lose 10 seconds in as many minutes upon being removed from the pocket and exposed, while sights are being taken, to the cold night air, and this variance may be concealed by the watch again varying its rate under fresh conditions of temperature and position ; so that, notwithstanding its vagaries, it may appear to have kept excellent time in the interval between any two observations taken to ascertain its correction. Nevertheless, in the observation of circum-meridian altitudes a small constant error in the time may

be kept from vitiating the result by arranging the observations as symmetrically as possible on each side of the meridian.

Instrumental and refraction errors should be eliminated by taking the mean results of observations of North and South stars on the same night, and at, as near as may be, similar altitudes.

Observations should not be extended beyond 30 minutes from the meridian.

It is a good plan to commence observing 15 minutes before the star culminates; allow yourself ample time and make the intervals between each altitude as uniform as convenient—continue observing till 15 minutes after the star has crossed the meridian. If you subsequently discover that you have taken many more observations on one side of the meridian than on the other, balance the series by a judicious discard.

Example 4.—On June 10, 1886, in assumed long. 32° 25′ 45″ E., lat. 34° 46′ 0″ N., barom. 28·25, therm. 76°, the following observations of *α Scorpii* (Antares) were taken with an artificial horizon. Watch 3m. 40s. fast of local mean time:—

2 Alts. Antares.	Time by Watch of Alts.	∾ Time by Watch of Transit	= Hour Angle Mean Time	= Equiv. Sid. Times.	No. extracted from Table XI.
° ′ ″	H. M. S.	H. M. S.	M. S.	H. S.	
58 4 20	10 46 5		11 43	11 45	271·2
58 9 5	10 50 20		7 28	7 29	109·9
58 11 15	10 53 55	10 57 48	3 53	3 54	30·0
58 12 5	10 50 45		57	57	2·1
58 10 30	11 3 5		5 17	5 18	54·8
58 8 5	11 5 45		7 57	7 58	124·5
58 5 45	11 8 25		10 37	10 39	222·7
7)407 1 5					7)815·2
				Mean No.	116·4

58 8 43·5	Mean inst. 2 alts.
37	– Index correction.
58 8 6·5	
25	– Eccentricity correction.
2)58 7 41·5	
29 3 50·7	Obsd. alt.
1 36	– Refraction (Table I.).
29 2 14·7	
1 38·1	+ Reduction.
29 3 52·8	Mer. alt.
90	
60 56 7·2	North M.Z.D.
26 10 39	South declination.
34 45 28	North latitude.

	H. M. S.
Star's R.A. . . .	16 22 27·85
Sid. time, noon . .	5 26 53·03
Approx. L.M.T. of transit	10 55 34·82
Long. E. in time (Table X.)	2 9 43
Approx. G.M.T. of transit	8 45 52

		H. M. S.
Sid. time at noon . .		5 26 53·03
Acceleration (Table IX.)	8h.	1 18·85
	45m.	7·39
	52s.	·14
Correct mean sun's R.A.		5 28 19·41
Star's R.A. . . .		16 22 27·85
L.M.T. of transit . .		10 54 8·44
Watch fast . . .		3 40
Time of transit by watch		10 57 48

		° ′ ″		
Assumed lat.	N.	34 46 0	cosine	9·914598
Declination	S.	26 10 39	,,	9·953002
Approx. M.Z.D. (sum)		60 56 39	cosec.	10·058415
Mean No. 116·4			log.	2·065953
Reduction 1′ 38″·1 = 98″·1			log.	1·991968

The difference between the "time by watch of transit" and the "time by watch of altitudes" gives the hour angle in mean time, which must be reduced to sidereal time by Table IX. The sidereal time thus found is the argument with which Table XI., giving the values of $\frac{2 \sin.^2 \frac{1}{2}\text{-hour angle}}{\text{Sin. } 1''}$, is entered for the extraction of numbers, the log. of the mean of which must be added to the log. cosine of the assumed latitude, the log. cosine of the declination and the log. cosec. of the approximate M.Z.D., the sum being the log. of the reduction in seconds, to be added to the true altitude.

When the mean sun's corrected right ascension is greater than the star's right ascension, increase the latter by 24 hours, to enable the former to be subtracted from it in deducing the L.M.T. of the star's transit.

If instead of a star the sun be observed, the watch time of *apparent* noon is computed, and the difference between this and the observed times by watch gives the hour angles in mean time, which latter should, in strictness, be reduced to its equivalent in apparent time, though in practice this is not necessary. The declination corresponding to the mean of the times of observation must be used.

By a single altitude near the meridian, the time being known.—It not unfrequently happens that a passing cloud prevents the meridian altitude being obtained, but if the time be known, and an observation of the sun or a star can be made when the number of *minutes* in the time from the meridian passage does not exceed the number of *degrees* in the meridian zenith distance of the object, the latitude can be deduced in the following manner:—

Example 5.—On June 9, 1886, in assumed longitude 32° 25′ 45″ E., and latitude 34° 46′ 0″ N., the barom. at 28·6 in., therm. 85°, the altitude of *a Scorpii* (Antares) in artificial horizon was observed to be 57° 54′ 40″ when watch, which was 3m. 48s. slow, showed 10h. 49m. 5s.

H. M. S.	
10 49 5	Watch time
3 48	,, slow
10 52 53	L.M.T. of observation

° ′ ″	
57 54 40	Instr. 2 alt.
37	– Index correction
57 54 3	
25	– Eccentricity
2)57 53 38	
28 56 49	Obsd. alt.
1 36	Refraction (Table I.)
28 55 13	True alt.

	H. M. S.
R.A. of star . .	16 22 27·8
Sid. time at noon . .	5 11 6·8
Approx. time of transit	11 11 21
E. long. in time (Table X.)	2 9 43
Approx. G.M.T. of transit	9 1 38

		H. M. S.
	Sid. time at noon	5 11 6·80
Table IX.) Acceleration	9h.	1 28·71
	1m.	·16
	38s.	·11
	Corrected R.A. of mean sun	5 12 35·78
	R.A. of star	16 22 27·80
	L.M.T. of transit	11 9 52
	,, observation	10 52 53
	Time from transit in mean time intervals	16 59 = Sidereal equivalent 17m. 1·79s.

	M. S.		
Time from transit	17 2	Rising (Tables)	2·43930
			168

	° ′ ″		
Assumed lat.	34 46 0	Cosine .	9·914598
Star's declination	26 10 39	Cosine .	9·953002
	No. 203·5	Log. . .	2·308580

True alt. 28° 55′ 13″ Nat. sin. 48353 7
5 5
48562 7 Nat. cosine

° ′ ″	
60 56 47	M.Z.D.N.
26 10 39	Dec. S.
34 46 8	Lat. N

Should the variance between the assumed and the deduced latitudes be great, the computation must be repeated, using the approximate latitude found by the observation.

By an altitude of the Pole Star at any known time.—This method is of great service in the northern hemisphere, as by it, with an artificial horizon, the latitude can accurately be found whenever the star is visible in any north latitude greater than about 12° and less than 60°; beyond these limits the star is either too near the horizon for its reflection to be seen in the quicksilver, or it is so near the zenith that its double altitude is out of range of the sextant.

With a clear sea horizon the observation can be made in nearly all north latitudes.

For the computation of the latitude, Tables are given in the Nautical Almanac, but in using them recourse must usually be had to interpolation, which entails nearly as much labour as the direct computation. Chauvenet,* in vol. i. p. 255, gives the formula:—

$$\phi = h - p \cos. t + \tfrac{1}{2} p^2 \sin. 1'' \sin.^2 t \tan. h,$$

where ϕ = the latitude,
h = the altitude,
p = the star's polar distance,
t = the hour angle.

Example 6.—On June 10, 1886, the altitude in an artificial horizon of *a* Ursæ Minoris (*Polaris*) was 67° 12′ 0″, when the watch, which was 4m. 8s. slow of L.M. time, showed 9h. 34m. 4s. Assumed longitude, 32° 25′ 45″ E.; barom. 29 in., therm. 75°.

* Chauvenet's "Astronomy," fifth edition, 2 vols. 8vo (Philadelphia).

By Nautical Almanac *Method.*

H.	M.	S.	
9	34	4	Time by watch
	4	8	Watch slow
9	38	12	L.M.T.
2	9	43	Assumed long. E. (Table X.)
7	28	29	G.M.T.

	H.	M.	S.
Sidereal time at noon . . .	5	15	3·36
(Table IX.) Acceleration { 7h. .		1	9·00
28m. .			4·60
29s. .			·08
R.A. of mean sun . . .	5	16	17
Local mean time . . .	9	38	12
Sidereal time of observation .. .	14	54	29

°	′	″		
67	12	0	2 alt. of Polaris.	
		37	Index correction.	
67	11	23		
		20	Eccentricity correction.	
67	11	3		
33	35	31	Observed alt.	
	1	21	Refraction Table I.).	
33	34	10	True alt.	
	1	0	Subtract(constant).	
33	33	10		
1	11	2	1st correctn. with argument, 14h. 54m. 29s.	
		6	2nd ,, ,,	{ 33° 34′ 10″ and 14h. 54m. 29s.
	1	11	3rd ,, ,,	{ June 10 and 14h. 54m. 29s.
34	45	29	N. latitude.	

By CHAUVENET.

	H. M. S.		
Local mean time	9 38 12		
Sidereal time mean noon . . .	5 15 3·36		
Reduction for 9h. 38m. 12s. (Table IX.) .	1 34·98		
	14 54 50·34		
Reduction for E. long. 2h. 9m. 43s. .	21·31		
Sidereal time of observation . . .	14 54 29		
R.A. of Polaris	1 17 1·6		
Hour angle W. .	13 37 27·4		
	24		
,, ,, E. . .	10 22 32·6	=	155° 38′ 9″
			180
			24 21 51

Declination N.

	88° 41′ 47″·8
	90
P.D.	1 18 12 = 4692″

Log. P.D. 4692 .	3·671358
,, Cosine 24 21′ 51″ .	9·959491
1° 11′ 14″) 4274″ log. .	3·630849 (1st term)

Log. (P.D. 4692)² . .	7·3427
,, Sine² 24 21′ 51″. .	9·2309
,, Tan. Alt. 33° 34 10″.	9·8219
,, ½ Sine 1″ . .	4·3845
″ log	0·7800 (2nd term)

True altitude (as above) .	33 34 10
1st term .	1 11 14
2nd ,, .	6
	34 45 30 N. lat.

[When the hour angle is less than 6h. or 90° the first term must be subtracted from the altitude instead of being added to it.]

TIME.

(*See Definitions of Apparent, Mean, and Sidereal Times.*)

By a set of altitudes of the sun.—Take an odd number of observations of one of the limbs of the sun when it bears as nearly as possible east or west, noting the time of each altitude by a watch. The mean of the times may, if the set has been promptly taken, be considered to correspond with the mean altitude; at the same time it must be remembered that it is only within reasonable limits that equal altitudes can be assumed to correspond to equal intervals of time. If, therefore, a series of observations extends over many minutes, either a correction for *second differences* must be made, or the series must be divided into groups, and separate deductions made from the mean of each group. Very low altitudes should be avoided on account of the uncertainty of refraction of objects seen near the horizon.

The local mean time of the instant of observation may be computed in the manner shown in the following example, and by comparing with it the mean of the times shown by the watch, the amount that the latter was fast or slow of the true time can be found.

Example 7.—On March 1, 1885, in long. 32° 25′ 45″ E., lat. 34° 46′ 0″ N., barom. 30 in., therm. 55°, the following altitudes of the sun's lower limb were observed in artificial horizon :—

H. M. S.	
4 49 36·6	Times by watch.
4 53 36·8	
4 57 22	
3) 14 40 35·4	
4 53 31·8	Mean of times.
2 9 43	E. long. in time (Table X.).
2 43 48·8	Approx. G.M.T.

° ′ ″	
24 42 50	2 Alt. ☉
23 9 30	
21 42 0	
3) 69 34 20	
23 11 27	Mean of alts. of ☉.
33	Index correction.
23 10 54	
15	Eccentricity correction.
2) 23 10 39	
11 35 19	Obsd. alt.
4 24	Refraction 4′ 33″ −Parallax 9″ (Tables I. and V.).
11 30 55	
16 10	☉ Semidiameter.
11 47 5	True alt. ☉ centre.
90	
78 12 55	Zenith distance.

Declination of ☉.

° ′ ″	
7 24 12·4	S. at Greenwich mean noon.
2 35·8	Var. in 2h. 44m.
7 21 36·6	S. at time of observation.

	″	
Var. of dec. in 1 hour =	57·07	decreasing.
×	2·74	H. time from noon.
	2 35· 8	

	° ′ ″				
Latitude . .	34 46 0 N. . . .		Log. secant . .	10·085402
Declination . .	7 21 37 S. . . .		,, ,, . .	10·003583
				10
Sum (different names)*	42 7 37			
Zenith distance	78 12 55			
Sum + Z.D.	120 20 32]	Half = 60° 10′ 16″	,, Sine . .	9·938258
				19
,, ~ ,,	36 5 18	,, 18 2 19	,, ,, . .	9·490759
				123
				2) 9·518154
		½-Hour angle = 35° 2′ 41″·6	Sine .	9·759077

35° 2′ 41″·6 × 2 = 70° 5′ 23″ =

H. M. S.	
4 40 21·5	(Table X.) apparent time of observation.
12 26·4	Equation of time.
4 52 47·9	Local mean time.
4 53 31·8	Mean of times by watch.
43·9	Watch fast.

	M. S.
E. of T. at noon .	12 27·85
Variation in 2h. 44m.	1·45
Reduced E. of T. .	12 26·4

If, when the observation is taken the sun is east of the meridian, the hour angle gives the time before apparent noon. Then, to obtain the *astronomical* reckoning, subtract the hour angle from 24 hours, and prefix the date previous to that of the civil day on which the observation is made.

* When the latitude and declination are of the same name—*i.e.*, both N. or both S., the *difference* of their values, instead of the *sum*, must be taken.

The error due to the apparent enlargement of the solar disc may be avoided by observing successively both limbs of the sun when they attain the same altitude, and noting the times, the mean of which corresponds to the instrumental altitude of the sun's *centre*. When this course is followed the reduction usually made by applying the semi-diameter is not required.

If the sun be rising, the altitude of the upper limb is first observed, and the time noted. The index of the sextant is not moved, but when the lower limb has risen to the same altitude, the time is again noted, and the sextant read off. When the sun is falling, the lower limb must first be observed.

For instance, suppose the index of the sextant is clamped at 87° 33′ 0″, and in an artificial horizon,

	H.	M.	S.
The sun's lower limb makes contact at	2	26	13
The sun's upper ,, ,,	2	29	24
2)	4	55	37
Mean . .	2	27	48·5

This mean is the time corresponding to the double instrumental altitude of the sun's centre, 87° 33′ 0″.

By a set of altitudes of a star.—The method of finding the hour angle of a star is similar to that of the sun, but this merely gives us the angular distance of the star from the meridian. To deduce from it the mean time, we must add it to or subtract it from the right ascension of the star (increasing the latter by 24 hours, if necessary), according to the position of the star west or east of the meridian, whence we obtain the "right ascension of the meridian." By subtracting from this the "right ascension of the mean sun," we obtain the angular distance from the meridian of the latter, which determines the local mean time. The "R.A. of the meridian" must be increased by 24 hours if it is less than the "R.A. of the mean sun."

Example 8.—On June 24, 1886, in assumed long. 32° 25′ 45″ E., lat. 34° 46′ 0″ N., the following altitudes of the star *α* Aquilæ (*Altair*) were taken in an artificial horizon, the star being east of the meridian, the barom. 29 in. and the therm. 75°:—

H. M. S.		° ′ ″	
10 40 6 10 44 36 10 47 55	Times by watch.	85 2 10 86 44 50 87 59 0	2 Alts.
3)32 12 37		3)259 46 0	
10 44 12·3	Means.	86 35 20	Double inst. alt.
2 9 43	Long. E. in time (Table X.).	20 −	Index correction.
8 34 29	Approx. G.M.T.	86 35 0	
		10	Eccentricity ,,
		2) 86 34 50	

		H. M. S.	° ′ ″	
Sidereal time at noon . .		6 10 15·16 (Almanac).	43 17 25	Observed alt.
(Table IX.) Acceleration	8h. .	1 18·85	58	Refraction (Table I.).
	34m.	5·59		
	29s. .	·08	43 16 27	True alt.
			90	
R.A. of mean sun . . .		6 11 39·68	46 43 33	Zenith distance.

	° ′ ″			
Latitude . . .	34 46 0 N.		Secant .	10·085402
Declination . .	8 34 15 N.		. .	10·004873
				5
Difference (same name)	26 11 45			
Zenith distance . .	46 43 33			
Diff. + Z.D. . . .	72 55 18	Half 36° 27′ 39″ . . .	Sine	9·773875
				111
,, ∼ ,, . . .	20 31 48	,, 10 15 54 . . .	,, . .	9·250282
				628
				2)9·115176
		½-Hour angle 21° 9′ 57″	Sine .	9·557588

21° 9′ 57″ × 2 = 42° 19′ 54″ = 2h. 49m. 19·6s. hour angle of star (Table X.).

	H. M. S.	
R.A. of star (from Almanac) . .	19 45 15·6	
Hour angle of star (east of meridian)	−2 49 19·6	
R.A. of the Meridian	16 55 56·0	
R.A. of mean sun (always −) . .	6 11 39·7	
	10 44 16·3	Local mean time.
	10 44 12·3	Mean of times by watch.
	0 0 4	Watch slow.

As the computation of the right ascension of the mean sun requires an approximate knowledge of the Greenwich mean time, which is generally deduced from the time shown by the watch, if the latter is found to have been

greatly in error, it will be necessary to recompute the right ascension of the mean sun, using the time given by the observation to find a more correct Greenwich date. In the solar observation, as the declination and the equation of time have to be reduced to the Greenwich time of observation, a similar course must be followed.

By equal altitudes of a star.—By this method all errors arising from imperfections of the sextant, from refraction errors constant for the period comprised by the complete observation, and from an incorrect estimate of the latitude, are avoided. A star is observed at any altitude (the nearer the prime vertical the better) east of the meridian, and again when it arrives at the same altitude west of the meridian, the times being noted. The mean between these times corresponds to the instant of the star's passage across the meridian; so, by comparing this mean with the true time of transit deduced from the star's right ascension the error of the watch can be found.

In practice, to obtain precision, three or more altitudes are observed east of the meridian, the sextant being set at each altitude, and the coincidence of the images perfected by the apparent motion of the star, the instrument in each observation being read off and the altitude noted, together with the time. Subsequently, when the star is west of the meridian, the index of the sextant is set successively, *in inverse order*, at the same altitudes, and the times of coincidence again noted. The mean of the times of the east altitudes and the mean of the times of the corresponding west altitudes are then treated simply as two observations of the same altitude.

The nearer the star's declination equals the latitude of the place (provided they are both north or both south) the shorter, within certain limits, may be the interval between the east and west observations. When the star passes very near the zenith it will be on or near the prime vertical (the most favourable position for observation)

shortly before and after the meridian passage; but the limited range of the sextant, when an artificial horizon is used, prevents an observation being taken within about 30° of the zenith; so practically the interval between the observations cannot be made very small. When the declination and latitude are of different names, it is necessary for accuracy to take the observations when the star is at a small altitude, and then of course the elapsed time is considerable.

Example 9.—On August 15, 1886, in long. 32° 25′ 45″ E. the following observations of *a* Aquilæ (*Altair*) were made at equal altitudes:—

East.		West.	2 Alt. Altair.
H. M. S.		H. M. S.	° ′ ″
7 51 15	Times by watch	12 21 28	98 51 20
7 52 45		12 19 59	99 21 20
7 54 15		12 18 30	99 51 20
3)23 38 15		3)36 59 57	
7 52 45	Mean of times	12 19 59	
12 19 59			
2)20 12 44			
10 6 22	Watch time of transit		

	H. M. S.
R.A. of Altair . .	13 45 15·9
Sidereal time at noon .	9 35 16
Approx. L.T. of transit .	10 9 59·9
Longitude E.	− 2 9 43
Approx. G.M.T. of transit	8 0 16·9

		H. M. S.
Sidereal time at noon		9 35 16
(Table IX.) Acceleration	8h.	1 18·85
	16s.	·04
	·9	·01
R.A. of mean sun		9 36 34·9

	H. M. S.
R.A. of Altair . .	19 45 15·9
R.A. of mean sun . .	9 36 34·9
Local M. T. of transit	10 8 41
Watch time of transit .	10 6 22
Watch slow . . .	2 19

By equal altitudes of the sun.—Owing to the change in the sun's declination which occurs in the interval between the east and west observations, the A.M. hour angle does not equal the P.M. hour angle when the altitudes are equal, as it does in the case of a star; it is therefore neces-

sary to apply to the mean of the A.M. and P.M. watch times a correction called *the equation of equal altitudes.* A table to facilitate the computation of this correction is to be found in most sets of Nautical Tables, but the following example is worked without such aid.

Example 10.—On Sept. 22, 1886 (civil reckoning), in long. 32° 25′ 45″ E., lat. 34° 46′ 0″ N., the following observations of the sun's lower limb were made at equal altitudes. (The times of both east and west observations are reckoned from noon of the same day—viz., Sept. 21.)

	☉ East.		☉ West.	2 Alt. ☉
	H. M. S.		H. M. S.	° ′ ″
Sept. 21	21 38 13	Watch times	26 19 15	84 9 0
	21 42 30		26 14 58	85 30 0
	21 46 50		26 10 41	86 51 0
	3)65 7 33		3)78 44 54	
	21 42 31	Means	26 14 58	
	26 14 58		21 42 31	
Interval	2) 4 32 27		2)47 57 29	
Half-interval	2 16 13·5	Middle Time	23 58 44·5	

E. long. 32° 25′ 45″ = in time (Table X.) 2h. 9m. 43s. E. = 2·15h. approx.

	° ′ ″
☉ Declination (p. I. N.A.) Sept. 22	0 14 48·3 N. (decreasing).
Corr. for hourly diff. 58″·46 × 2·15	2 5·6
Corrected declin.	0 16 53·9 N.
	90
Polar distance (increasing)	89 43 6·1

Hourly diff. in declination (N.A.) 58″·46
Half-interval (2h. 16m. 13s. = 2·27h.) × 2 ·27

	132″·7 . . .	Log.	2·122871
Lat. . . .	34° 46′ 0″ N. . .	Tan.	9·841457
Half-interval 2h. 16m. 13s. =	34° 3′ 15″ . .	Cosec.	10·251830
	First term 164″·5 . .	Log.	2·216158

132″·7	. . . .	Log.	2·122871
Declin. 0° 16′ 54″ N.	. . . .	Tan.	7·690071
Half-interval 2h. 16m. 13s. = 34° 3′ 15″	.	Cotan.	10·170127
Second term ·9	. . .	Log.	9·983069

When the lat. and decl. are the same name, subtract second term from first term.
" " " contrary names, add " to "

First Term 164″·5
Second " ·9

163·6 ÷ 15 (constant) = 10″·9 equation of equal altitudes.

		H. M. S.
Middle time by watch	. . . . . .	23 58 44·5
Equation of equal altitudes	{ P.D. increasing + / " decreasing − }	+ 10·9
Watch time at apparent noon	. . . . .	23 58 55·4
M.T. at app. noon (24h. − E. of T. 7m. 18·5s.)	.	23 52 41·5
Watch fast of local mean time	.	0 6 13·9

It will be observed that the computation requires that the latitude and longitude of the place of observation be approximately known, but one of the advantages of this method of determining time is that both the assumed latitude and longitude may differ considerably with the truth without an appreciable error being introduced in the deduced time.

Erroneous index correction, eccentricity, or imperfect graduation of the sextant do not influence the result, as the sextant reading is used merely for the purpose of setting the index at the same altitude in the P.M. observations as it indicated in those taken in A.M., and then only when more than one altitude is taken on each side of the meridian.

If it is not intended to observe more than two or three altitudes, it is well to clamp the index, and let it remain fixed while the times of contact with their images, of the sun's upper and lower limbs in the A.M. sights, and of the lower and upper limbs in the P.M. sights, are noted accord-

ing to the method explained at page 51. In this way two observations (in which the mean of the times corresponds with the altitude of the sun's centre) are obtained on each side of the meridian, and while no reading of the instrument is necessary, all chance of error through setting the index incorrectly, when the west sights are taken, is avoided.

A convenient method of finding the *rate* of a watch without determining its error is by observing a star on different evenings at the time that it attains the same altitude on the same side of the meridian.

The interval of time between a star's successive appearances at any given altitude is 24 hours sidereal time, or 23h. 56m. 4·0906s. mean solar time. The star, therefore (provided its declination and R.A. remain constant), appears at the same altitude 3m. 55.91s. (nearly) earlier every evening; so that if the watch which has been used to note the time of the observations of the star gains or loses, its error is at once apparent, and by dividing this error by the number of days between the observations the daily rate of the watch is determined.

Example 11.—On June 9, 1886, the 2 alt. of *Antares* was $57° 54' 40''$ at 10h. 49m. 5s. On June 15, the star was at the same altitude at 10h. 24m. 16·5s., the time being taken with the same watch at both observations.

			H. M. S.
June 9	.	Time by watch .	10 49 5
,, 15			
Interval 6 days		3m. 55·91s. × 6 —	23 35·5
			10 25 29·5
June 15	.	Time by watch .	10 24 16·5
		Loss in 6 days .	1 13
		Daily rate, losing .	12·1

During the interval the changes in the star's decli-

nation and R.A. will generally be so small that they may be neglected; but if the increase in R.A. is found by the Almanac to be appreciable, add it to the time of the second observation.

Both observations must be made at the same place, as a change of latitude would alter the time of the stars attaining the given altitude, while a change of longitude would, if not allowed for in the computation, produce an error equal to the difference between the local times of the two places at which the observations were made.

LONGITUDE.

(*See Definition.*)

By chronometers.—The difference of longitude, in time, of any two places, is found by comparing their local times at the same absolute instant. This difference in time is reduced to degrees, &c., of arc by simple proportion, 15° being equivalent to 1 hour (see Table X.). If, therefore, the local time at Greenwich (or other place, the longitude of which is known) be ascertained by chronometer or other means, at the same instant that the local time of the observer's station is found by any of the methods already explained, the longitude can easily be determined.

The chief difficulty is to obtain accurate Greenwich time, which at sea, and when circumstances permit on land, is generally found by means of chronometers, whose error on Greenwich mean time at a certain date and whose daily rate have been precisely ascertained.

Example 12.—On March 1, 1885, in assumed longitude 32° 25′ E., lat. 34° 46′ 0″ N., the double altitude of the sun's lower limb was observed to be 23° 11′ 27″ when the chronometer showed 2h. 41m. 50·5s.; the chronometer had on Feb. 3, 1885, been found to be 1m. 27s. slow of Greenwich mean time, and its rate ·5s. gaining.

(The local mean time, when the sun's lower limb

attained, on March 1, 1885, at this station an altitude of 23° 11′ 27″, was found in Example 7 (see page 50) to be 4h. 52m. 47·9s.).

	H. M. S.	H. M. S.
Local mean time . .		4 52 47·9
Chronometer time . .	2 41 50·5	
,, slow, Feb. 3, '85 +	1 27	
	2 43 17·5	
Accumulated rate, 26 days at ·5s. daily gain . .	13	
Greenwich mean time, March 1, '85 .		2 43 4·5
Longitude east of Greenwich . .		2 9 43·4 = 32° 25′ 51″ (Table X.)

It is evident that in cases where the rate of the chronometer has not been verified for a considerable period, large errors in the longitude may be introduced by any derangement of the chronometer rendering the "accumulated rate" incorrect.

Unfortunately, the chronometer which will maintain a sufficiently uniform rate, when travelling by land, to be implicitly trusted, has yet to be invented.

By lunar distances.—By deducing from an observation (made in the manner explained at page 29), the moon's true angular distance from the sun, a planet, or a star lying in or near the moon's path, and comparing it with the distances registered under the corresponding times in the Nautical Almanac, the Greenwich mean time at the instant of observation can be determined. The moon's motion in her orbit is about 13° daily, or approximately at the rate of 1′ in 2 minutes of time; if, therefore, her distance from the sun, a planet, or a "lunar distance star," can be ascertained within 10″, the corresponding Greenwich time will be known within 20 seconds, and hence the longitude within 5′.

The longitude in time (which can be converted into degrees, &c., by simple proportion or by Table X.) is the difference between the Greenwich mean time thus

obtained and the local mean time ascertained either by an observation made simultaneously with the measurement of the lunar distance, or by a watch whose error must be found shortly before or after the lunar observation.

The longitude is east or west according as the Greenwich time is less or greater than the local time.

The apparent distance of the moon's centre (the instrumental distance of the limb, or limbs, corrected for index error and eccentricity with the semi-diameters of the observed bodies applied) must, before it can be compared with the distances tabulated in the Almanac, be reduced to what it would have been if the observation had been made at the earth's centre, to which the true distances are referred.

This process, called "clearing the distance," is rendered necessary by the effects of parallax and refraction, which to an observer on the earth's surface cause the moon to appear lower, and the sun, planets, and stars higher than they would if seen from the earth's centre.

The Nautical Almanac gives for every third hour of Greenwich mean time the true distances of the apparent centre of the moon from the sun's centre, and from the following planets and stars whenever they are available for the determination of longitude.

		Right Ascension.	Declination.
Saturn, Jupiter, Mars, Venus	Planets.		
		° ′ ″	° ′ ″
α Arietis		2 0 48	22 55 38 N.
α Tauri	(*Aldebaran*)	4 29 27	16 16 45 N.
β Geminorum	(*Pollux*)	7 38 25	28 17 43 N.
α Leonis	(*Regulus*)	10 2 21	12 31 3 N.
α Virginis	(*Spica*)	13 19 14	10 34 8 S.
α Scorpii	(*Antares*)	16 22 27	26 10 36 S.
α Aquilæ	(*Altair*)	19 45 14	8 34 20 N.
α Piscis Australis	(*Fomalhaut*)	22 51 23	30 13 26 S.
α Pegasi	(*Markab*)	22 59 7	14 35 53 N.

The time, corresponding to any distance falling between those distances registered in the Almanac, must be obtained by interpolation, to facilitate which the proportional logarithms of the differences of the distances, at intervals of three hours, are also given in the Almanac. Since an error of 10″ in the distance causes the deduced Greenwich time to be incorrect to the extent of about 20 seconds, it is essential, in determining even a fair approximation of the longitude, not only that the observation itself should be most carefully made, and the computations carried out with the greatest precision, but also that every available means be employed to eliminate or reduce to a minimum all errors of observation and of the instrument; and, further, that care be taken to avoid the introduction of errors which (in cases where an unfavourably situated lunar-distance star has injudiciously been employed) may arise in clearing the distance by means of the Tables, which are invariably used to avoid the tedious method of calculating the true distance by spherical trigonometry.

Under these circumstances a single set of distances measured only on one side of the moon cannot be trusted to give a very accurate value of the longitude, although an observer, who has by experience gained a knowledge of the usual difference between his longitudes of the same place deduced from observations, made at various distances and of stars east and stars west respectively, will be able to obtain a closer approximation to the true longitude than he would be able to do were he unacquainted with the values of the errors to which his determinations are severally liable.

Instrumental errors will be nearly eliminated by taking the means of longitudes deduced from observations of stars both east and west of the moon, especially if the distances are equal or nearly so.

Constant errors of observation such as arise from the habit of always making the contact of the observed bodies

imperfectly, either by bringing the star's light too far on to the moon's disc, or by leaving a small space between the star and the moon's limb, will be eliminated by observing stars on opposite sides of the moon *and measuring the distances from her opposite limbs*, but as only the enlightened limb of the moon is available, it is impossible to make both observations on the same night; one of them must be postponed till the moon has travelled in her orbit sufficiently to cause the solar rays to enlighten the opposite limb to that first observed; or, in other words, if the first observation be made before the time of full moon, the second must be made when the moon is waning, and *vice versâ*.

When an observation on one side of the moon has been made, a star on the opposite side which has about the same lunar distance should be sought; if it has nearly the same proportional logarithm opposite to it in the Almanac, it will be an advantage, as under these conditions the velocity of the moon from and towards the two stars will be equal.

The greater the velocity of the moon from or towards a star, the greater the precision with which an observation may be made, and as a comparatively small proportional logarithm indicates a rapid change in the lunar distance, those stars which have the least proportional logarithm should (other things being equal) be selected for observation; at the same time a star which is at a low altitude must not be chosen, on account of the uncertainty of the refraction of objects near the horizon.

Lunar distances less than 45° or more than 100° should not be employed. In very small distances sensible error is introduced by the effect which refraction produces in distorting the disc of the moon, and that of the sun if it be employed, when the altitude is less than 45°.

The following table exhibits the difference between the true Greenwich time and that deduced from 32 lunar observations made at various distances on both sides of

the moon. It will be observed in the observations made of stars east of the moon that the error generally increased as the lunar distance decreased :—

Star or Planet.	Proportl. Logarithm.	Lunar Distance.	Error of Lunar in the Deduced Greenwich M.T.	
			Star East of Moon.	Star West of Moon.
		°	H. M. S.	H. M. S.
Spica .	2942	21	......	4 Slow
Antares .	3357	25	......	1 ,,
,, .	3157	28	1 16 Fast	
Aldebaran .	2565	29	1 38 ,,	
Antares .	3392	32	1 29 ,,	
α Arietis .	2915	41	......	55 ,,
Antares .	3181	45	......	0 0 0
,,	2381	45	......	0 0 0
Regulus .	2364	45	......	41 ,,
Fomalhaut	3466	46	55 ,,	38 Fast
Aldebaran .	2147	48	......	16 Slow
Fomalhaut	3707	51	51 ,,	
Pollux .	2422	53	......	1 7 ,,
Fomalhaut	3241	54	......	24 Fast
Spica .	3050	55	......	50 Slow
Altair .	3449	58	......	16 ,,
Fomalhaut	3590	59	1 7 ,,	
Saturn .	2840	60	1 25 ,,	
α Arietis .	3068	61	49 ,,	
Antares .	2736	61	43 ,,	
Spica .	2248	62	48 ,,	
Jupiter .	2285	67	1 3 ,,	
Antares .	2203	77	13 Slow	
Aldebaran .	2009	78	......	56 ,,
Pollux .	2128	89	......	58 ,,
Altair .	2960	95	......	17 Fast
Jupiter .	2401	96	10 ,,	
Aldebaran .	2917	98	3 ,,	
Regulus .	2657	99	......	6 ,,
Pollux .	2441	100	......	44 Slow
Saturn .	3061	109	52 Fast	
Aldebaran .	2985	110	15 ,,	

By rejecting the six observations of distances less than 45°, which in the determination of longitude should not be employed, we find that the mean error by the star-east distances is 38·6s. fast, and by the star-west distances 20·2s. slow, while the mean of the whole twenty-six observations gives the Greenwich mean time within 9·2 sec. and the longtitude within 2′ 18″.

As, in clearing the distance, the altitudes of the star and moon at the time that the distance was measured must be known, they must either be taken by assistants working simultaneously with the observer, or he must himself ascertain the altitudes both before and after he has measured the distances, and by interpolation find what they were, at the mean of the times corresponding to the mean of the distances (see Example 21, p. 81); this course cannot be followed, however, if either of the bodies crosses the meridian between the first and second observation of its altitude.

In cases where the latitude and the local mean time have been very accurately ascertained, and where the assumed longitude is not in error to any great extent, so that the declination and right ascension of the moon can be correctly obtained from the Almanac, the altitudes may be computed instead of being observed. This method may be employed with advantage after the longitude has once been approximately determined, as time in which to observe a greater number of distances is thereby gained.

The observations, the times of each being noted, should be made in the following order, if there be only one observer:—

1. Altitude for time star east.
2. Altitude for time star west.
3. Altitude (not less than 15°) of lunar-distance body farthest from the meridian.
4. Altitude of lunar-distance body nearest to the meridian.
5. Five, seven, or more distances.
6. Altitude of lunar-distance body nearest to the meridian.
7. Altitude of lunar-distance body farthest from the meridian.
8. Altitude for time star west.
9. Altitude for time star east.
10. Barometer and thermometer.

If the altitudes of the moon and lunar-distance star are to be computed, of course Nos. 3, 4, 6, and 7 will be omitted, but then the latitude, if not known, must be accurately determined.

There are numerous methods of clearing the distance, most of them requiring special Tables, which are published together with instructions for their use. These methods, though they are not all rigorously accurate, will, when the atmosphere does not differ much with its mean state, and if a favourably situated lunar-distance star has been employed, generally give the true distance within 3″ or 4″, which is practically sufficiently exact in relation to the degree of accuracy attainable in sextant observations. The following example, in which the altitudes are computed, is worked by the method of natural versed sines.

Example 13.—On May 18, 1886, in assumed long. 32° 25′ 48″ E., lat. 34° 46′ N., the following distances between *Regulus* and the far limb of the moon were observed, the times being noted by a watch whose error on local mean time had been found to be 3m. 45s. slow. *Regulus* west of moon. Bar. 29·5 in., therm. 50°.

	Times by Watch.		Lunar Distances.	
	H. M. S.		° ′ ″	
	11 6 5		99 55 20	
	8 50		99 56 35	
	13 20		99 57 45	
	16 15		99 59 0	
	20 15		100 0 10	
	5) 64 45		5) 499 48 50	
	11 12 57	Mean of times.	99 57 46	Mean of distances.
	3 45	Watch slow.	41	Index correction.
	11 16 42	Local mean time.	99 57 5	
(Table X.)	2 9 43	E. long. in time (assumed).	15 37	Moon's augmented semi-diam. (far limb).
	9 6 59	Approx. G.M.T.	99 41 28	Apparent distance.

	° ′ ″			H. M. S.
Moon's declination at 9h.	16 39 0·6	S.	Moon's right ascension at 9h.	16 26 34·12
,, variation in 6m. 59s.	34·2		,, variation in 6m. 59s.	15·4
Reduced declination .	16 39 35	S.	Reduced R.A. . . .	16 26 50

	′ ″		′ ″
Moon's horizontal parallax at noon	56 52·9	Moon's semi-diam. at noon .	. 15 31·5
,, variation in 9h. 7m. . .	13·6	,, variation in 9h. 7m.	. 3·8
	56 39		15 28
Correction for latitude (Table VIII.)	4	Augmentation for alt. 34° (Table VII.) . . .	9
Reduced horizontal parallax .	56 35	Moon's augmented semi-diam.	. 15 37

COMPUTATION OF MOON'S ALTITUDE.

	H. M. S.		
Sidereal time at noon . . .	3 44 22·57	(Almanac.)	
Acceleration for 9h. 6m. 59s. . .	1 29·86	(Table IX.)	
Mean sun's R.A.	3 45 52		
Local mean time (as above) . .	11 16 42		
R.A. of meridian	15 2 34		
Moon's R.A.	16 26 50		
Difference . .	1 24 16	Rising (Tables)	3·82502
Latitude 34° 46′ 0″ N.		Cosine .	9·914598
Moon's declination 16° 39′ 35″ S.		Cosine .	9·981377
		* Nat. number 5260·1 Log.	3·720995

	° ′ ″		
Sum (lat. and dec. diff. names)	51 25 35	Versed sine	376348
			134
Zenith distance . .	55 11 9	Versed sine	429083
	90		
Moon's true altitude .	34 48 51		
"Moon's correction" parallax−ref.	45 27	(Sin. parallax = sin. hor. par × cos. alt.)	
Moon's apparent alt.	34 3 24		

COMPUTATION OF ALTITUDE OF *REGULUS.*

	H. M. S.		
R.A. of meridian (as above)	15 2 34		
Regulus R.A. (Almanac) .	10 2 19		
Difference .	5 0 15	Rising	4·87054
Lat. 34 46 0 N.		Cosine	9·914598
Star's dec. 12 31 17 N. . . .		Cosine	9·989545
		* Nat. number 59522·7 Log.	4·774683

	° ′ ″		
(Lat. and dec. same name) diff.	22 14 43	Versed sine	07434 9
			8 1
Zenith distance	70 42 38	Versed sine	66965 7
	90		
Star's true alt.	19 17 22		
Refraction +	2 41	(Table I.)	
Star's appt. alt.	19 20 3		

* This "natural number" should always be taken out to one place of decimals, so as to enable the versed sine to be taken to six figures.

COMPUTATION OF THE "TRUE DISTANCE."

Moon's Correction (Tables).

			Correction.		Auxiliary Arc.
		° ′ ″	° ′ ″		° ′ ″
Arguments	Moon's app. alt.	34 3 24	0 45 27	. . .	60 17 20
	,, hor. para.	56 35		Alt. of star 19°	0
				(B)	60 17 20

	° ′ ″
Star's apparent altitude . .	19 20 3
Moon's ,, ,, . . .	34 3 24
Sum of apparent altitudes (A) .	53 23 27
Apparent distance (as above (C)	99 41 28
Sum of apparent altitudes . .	53 23 27
Moon's correction + 45′ 27″ / Star's refraction − 2 41 .	42 46
Sum of true altitudes . . .	54 6 13
	180
Sum of true zenith distances .	125 53 47

	° ′ ″				
A + B . . .	113 40 47	Versed sine	1·401415	Parts for seconds	209
A ∼ B . . .	6 53 53		007208		30
B + C . . .	159 58 48		1·939494		82
B ∼ C . . .	39 24 8		227266		24
Sum of true zenith distances .	125 53 47		1·586137		186
			531	Sum of parts	531
True distance	99 19 33	Versed sine	162051		

COMPUTATION OF THE LONGITUDE.

	° ′ ″		
True distance	99 19 33		
Distance at 9h. by Nautical Almanac . .	99 15 39	. Pro. log. of diff.	2657
Difference . .	3 54	. Pro. log. ,,	1·6642
	H. M. S.		
Proportional part of time	0 7 11	. Pro. log ,,	1·3985
Time of distance by N. Almanac . . .	9		
	9 7 11		
Correction for 2nd diff. (Table in N. A.)	− 1		
Greenwich mean time by lunar . . .	9 7 10		
Local mean time by corrected watch . .	11 16 42		
Longitude east of Greenwich . . .	2 9 32 = 32° 23′ 0″ E. (Table X.)		

By the moon's altitude.—As the Greenwich mean time corresponding to the moon's right ascension is registered in the Nautical Almanac for every hour throughout the year, if the right ascension be determined by means of an observation of the moon's altitude, the Greenwich mean time at the instant of observation, and thence the longitude, can be deduced.

This method will not generally give such reliable results as that of lunar distances, and should only be employed in low latitudes where the moon can be observed at once on or near the prime vertical, and at such an altitude that the correction for refraction can be accurately ascertained.

Shortly before the moon's altitude is observed the error of the watch on local mean time should be determined by observing the altitude of a star on or near the prime vertical (see Example 8), and on the same side of the meridian as the moon. Extreme accuracy both in the observations and throughout the computation is absolutely requisite, and the *mean* longitude deduced from observations of the moon, both east and west of the meridian, should be adopted. If time permits, it is well to take observations before and after full moon.

If the assumed longitude employed in determining the approximate Greenwich time is found to be greatly in error, it will be necessary to repeat the computation, employing the more correct Greenwich time deduced from the observation of the moon's altitude, for correcting the elements taken from the Almanac.

Example 14.—On June 17, 1886, in assumed lat. 34° 46′ 0″ N., long. 32° 30′ 0″ E.; barom. 29·5 in., therm. 75°, the following observations were made of the altitude of the moon's lower limb when she was east of the meridian, the watch being 7 sec. slow of local mean time, and an artificial horizon being used.

Times by Watch.

H. M. S.	
9 55 31·6	
9 59 30	
10 4 17·5	
3) 29 59 19·1	
9 59 46·4	Mean of times.
7	Watch slow.
9 59 53·4	Local mean time.
2 10 0	E. long. in time (Table X.)
7 49 53·4	Approx. G.M.T.

	′ ″
Moon's semi-diam. at noon	15 4·3
Variation in 7h. 50m.	2·3
	15 2
Augmentation (Table VII.)	5
Moon's augmented S.D.	15 7

	° ′ ″
Moon's dec. at 7h.	18 42 30 S.
Variation for 49m. 53s.	50
Reduced declination	18 41 40 S.

Double Altitudes of ☽

° ′ ″	
39 1 50	
40 11 20	
41 34 20	
3) 120 4 30	
40 15 50	Mean of double altitudes
20 −	Index correction.
40 15 30	
23 −	Eccentricity correction
2) 40 15	
20 7 33	Obsd. alt. ☽
15 7	Augmented S.D. of ☽
20 22 40	Obsd. alt. ☽'s centre.
49 10	Parallax in alt.—Refraction. { See "Observations of the Moon," page 86.
21 11 50	True alt.
90	
68 48 10	Zenith distance.

	° ′ ″
Moon's hor. parallax at noon	55 12 9
Variation in 7h. 50m.	8 1
	55 4 8
Correction for latitude (Table VIII.)	3 8
Reduced hor. parallax	55 1 0

		H. M. S.			
Sidereal time at noon		5 42 39·26			
(Table IX.) Acceleration { 7 hours		1 9·00			
49 min.		8·05			
53 sec.		·15			
R.A. of mean sun		5 43 56·46			
Local mean time		9 59 53·4			
Sidereal time of observation		15 43 49·86			
Latitude	34 46 0N.			Sec.	10·085402
Declination	18 41 40 S.			,,	10·023539
Sum (diff. names)	53 27 40				
Zenith distance	68 48 10				
Sum + Z.D.	122 15 50	Half 61 7 55		Sin.	9·942372
,, ~ ,,	15 20 30	,, 7 40 15		,,	9·125421
					2) 9·176734
			° ′ ″ 22 48 15	Sin.	9·588367
			2		
				H. M. S.	
Moon's hour angle (east)			45 36 30 =	3 2 26	
Sidereal time of observation (as above)				15 43 49·86	
Computated R.A. of moon *				18 46 15·86	
Moon's R.A. per Almanac at 7 hours (next less R.A.)				18 44 25·9	
			Diff.	1 49·96	
Constant		4771			
Pro. log. of diff. 1m. 49·9s.		1·9920			
		2·4691			
Pro. log. of 2m. 10s.		1·9195			
				H. M. S.	
		5496	Pro. log.	0 50 47	

	H. M. S.
Next less R.A. of moon at 7h.	18 44 25·9
Next greater ,, 8h.	18 46 36
Diff.	2 10·1

	H. M. S.	
Nautical Almanac hour	7	(Corresponding to next less R.A.).
G.M.T. of computed R.A.	7 50 47	
Local mean time	9 59 53	
Longitude, east	2 9 6 =	° ′ ″ 32 16 30 E. (Table X.)

* When the moon is west of the meridian, the hour angle must be subtracted from the sidereal time (increasing the latter by 24 hours if necessary to obtain the moon's R,A.

In the above example, as the moon was not very favourably situated, the computed Greenwich mean time was 37 sec. fast of truth, and consequently the deduced longitude was 9′ 15″ too far west.

As the station was in north latitude, better results would have been obtained had the observation been made either earlier or later in the month, when the moon's declination was north instead of south; and further, as the moon's motion in declination was from south to north, an observation taken west instead of east of the meridian would have been more accurate, since the most favourable position of the moon is when she is descending from the meridian if her motion in declination is towards the observer, and when she is rising if her motion in declination is from him

THE VARIATION OF THE COMPASS.

By a single altitude.—The variation or declination of the compass is the angle included between the terrestrial and magnetic meridians. The *error* of the compass is the total effect of the variation and deviation or local attraction, and is the angle through which the needle is deflected from the true meridian.

The error can be determined by comparing the bearing of a heavenly body with the azimuth of the latter deduced from an altitude observed simultaneously with the compass bearing.

The observed body should be as far from the meridian as possible, and the means of several bearings, and of the corresponding altitudes, should be employed.

Example 15.—On Oct. 25, 1886, in latitude 34° 46′ N., the following compass bearings and double altitudes of β Orionis (*Rigel*), east of the meridian, were observed Barom. 30 in., therm. 60°.

	Double Altitudes.		Compass Bearings.
	° ′ ″		° ′ ″
	45 30 0		125 40 0
	46 54 0		126 10 0
	48 28 0		127 0 0
	3) 140 52 0		3) 378 50 0
Mean of 2 alts. . .	46 57 20	Star's mean bearing	126 16 40
Index correction .	20		
	46 57 0		° ′ ″
Eccentricity correction	25	Declination of *Rigel*	8 20 0 S.
	2) 46 56 35		90 0 0 + (Lat. N.)
Observed alt. . .	23 28 17	Polar distance	98 20 0
Refraction (Table I.).	2 11		
True altitude . .	23 26 6 . .	Secant	10·073657
Latitude . . .	34 46 0 . .	,,	10·085402
Polar distance . .	98 20 0		
Sum	2) 156 32 6		
Half-sum . . .	78 16 3 . .	Cosine	9·308229
P.D. ∾ half-sum . .	20 3 57 . .	,,	9·972804
			2) 9·440092
	° ′ ″ 31 39 32		
	× 2 . .	Sine	9·720046
True azimuth (south)	63 19 4 (towards east)		
	180		
,, bearing* .	116 40 56 (from north)		
Compass bearing .	126 16 40		
Compass error .	9 35 44 W.		

[The true azimuth is reckoned from the south in north latitude, and from the north in south latitude, towards the east when the altitude is increasing, and towards the west when it is decreasing.]

Of course, not only a star, but any heavenly body can be used, provided an approximate knowledge of the Greenwich mean time can be obtained, so that the elements taken from the Nautical Almanac may be corrected; the computation of the compass error is then carried on as in the above example, after the true altitude has been deduced.

It is not necessary to observe the altitude if the local mean time is accurately known, as the true azimuth can be determined by means of the hour angle. When this

* *Bearings* are reckoned from the north (360° or 0°), through the east (90°), south 180°), and west (270°) back to north.

method is employed a star near the Pole will give the most accurate results, especially if it is observed at its greatest elongation.

Example 16.—In assumed long. 32° 25′ 48″ E., lat. 34° 46′ 0″ N., the compass bearing of *a* Canis Majoris (*Sirius*), was 126° on Oct. 25, at 12h. 9m. 30s., by a watch which was 2m. 12s. fast of L.M.T. Star east of meridian.

H. M. S.	
12 9 30	Time by watch.
2 12	Watch fast.
12 7 18	Local M.T.
2 9 43	Long. in time E. (Table X.).
9 57 35	Approx. G.M.T.

	H. M. S.
Sid. time at noon . . .	14 15 11·28
(Table IX.) Acceleration { 9h. .	1 28·71
57m. .	9·36
35s. .	·10
R.A. of mean sun . . .	14 16 49
Local mean time	12 7 18
R.A. of meridian . . .	26 24 7
R.A. of *Sirius*	6 40 10
Hour angle (west) . . .	19 43 57
	24
,, ,, (east) . . .	4 16 3

Star's declination 16° 33′ 40″ S.	Tan.	9·473303
,, hour angle 4h. 16m. 3s. = 64° 0′ 45″	Sec.	10·358352
* 34° 9′ 48″	Tan.	9·831655

	° ′ ″		
Star's hour angle	64 0 45	Tan.	10·312058
*	34 9 48	Cosin.	9·917736
(Sum if lat. and dec. are of different names) Latitude N. 34° 46′ 0″ }	68 55 48	Cosec.	10·030052
(Difference ,, ,, same ,,) * + 34 9 48 }			
Star's true azimuth from south	61 12 3	Tan.	10·259846
	180 0 0		
,, true bearing from north	118 47 57		
Compass bearing	126 0 0		
Compass error	7 12 3 W.		

By equal altitudes.—The declination of a star being practically constant for short intervals of time, equal altitudes correspond to equal azimuths. If the compass bearing of a star which passes south of the zenith be observed when it is at the same altitude on opposite sides of the meridian, the difference between the mean of the two magnetic bearings and 180° gives the error of the compass, west or east according as it is greater or less than the true bearing of the south pole.

This method requires no knowledge of the latitude, longitude, or time; while instrumental errors of the sextant do not influence the result.

Example 17.—The compass bearing of a star east of the meridian was found to be 174° 30′, when the altitude (double) observed in an artificial horizon was 46° 22′ 0″. When the star descended to the same altitude west of the meridian the compass bearing was 193° 15′.

	°	′	″
East compass bearing	174	30	0
West ,, ,,	193	15	0
	2) 367	45	0
Mean ,, ,,	183	52	30
True bearing of south	180	0	0
Compass error	3	52	30 W.

When a star passing north of the zenith is employed, the difference between the mean of the compass bearings and 360°, or zero, determines the compass error. Equal hour angles may be employed instead of equal altitudes, if the local mean time is known.

Example 18.—On Oct. 30, 1886, in assumed longitude 32° 25′ 45″ E., the compass bearing of α Piscis Australis (*Fomalhaut*) was 174° 30′, when the watch, which was 4m. 7s. fast of local mean time, showed 7h. 12m. 53s. A second bearing of the same star was found to be 193° 15′ at the time when its west hour

angle equalled its east hour angle, this time having been computed in the following manner :—

	H. M. S.
R.A. of *Fomalhaut* . .	22 51 24
Sidereal time at noon .	14 34 54
Approx. local time of transit . .	8 16 30
Long. in time, east (Table X.) . .	2 9 43
Approx. G.M.T. of transit	6 6 47

		H. M. S.
Sidereal time at noon .		14 34 54·05
Acceleration (Table IX.)	6h. .	59·14
	6m. .	·99
	47s. .	·13
R.A. of mean sun .		14 35 54

		H. M. S.
R.A. of *Fomalhaut*		22 51 24
R.A. of mean sun		14 35 54
Local mean time of transit . .		8 15 30
,, ,, ,, 1st compass bearing		7 8 46
Hour angle of *Fomalhaut* (east) .		1 6 44
Local mean time of transit (as above)		8 15 30
,, ,, ,, 2nd compass bearing		9 22 14
Error of watch	+	4 7
Proper time by watch to observe 2nd compass bearing . .		9 26 21

	H. M. S.
Watch time of 1st bearing	7 12 53
Error of watch (fast) .	4 7
Local mean time . .	7 8 46

	H. M. S.		° ′ ″
Compass bearing at	7 12 53	by watch	174 30 0
,, ,,	9 26 21	,,	193 15 0
			2)367 45 0
Mean bearing . . .			183 52 30
True bearing of south .			180 0 0
Compass error . . .			3 52 30 W.

When the sun is employed in determining the compass error by equal altitudes it is necessary to apply a correction for the change of the declination, and to deduce this correction, the times of both east and west observations must be noted.

Example 19.—On Oct. 31, 1886, in lat. 34° 46′ 0′ N., the following compass bearings of the sun, and observations of its lower limb at equal altitudes (double 51° 39′ 55″), were observed, the times of both east and west observations being reckoned from noon of the same day—viz.,

Oct. 30, the civil day previous to the one on which the observations were taken.

Times by Watch.	H. M. S.	Compass Bearings. ° ′ ″	
Sun east, Oct. 30	20 49 39	139 30 0	
,, west ,,	26 37 28	228 25 0	
Interval . .	5 47 49	367 55 0	
Half-interval .	2 53 54	183 57 30	Mean.
		4 9	Correction (always –).
		183 53 21	Compass reading of meridian.

Hourly variation in declination (N.A., page 1) 48″ 59
Interval 5h. 47m. 49s. = 5·8h. . . . × 5·8

281·8 Log. 2·449941

		° ′ ″		
Latitude		34 46 0	Sec.	10·085402
Half-interval	2h. 53m. 54s. =	43 28 30	Cosec.	10·162388
	8′ 18″·6 =	498″·6	Log.	2·697731
Correction (half)	4 9·3			

	° ′ ″	
Compass bearing of meridian	183 53 21	
True ,, ,,	180	
Compass error	3 53 21	W.

THE DIRECTION OF THE MERIDIAN LINE.

(*See Definition of Meridian Line.*)

By the angular distance of a referring mark from the sun.—By determining the azimuth of the sun, and the difference of azimuths of the sun and a referring mark at the same instant, the azimuth of the mark becomes known, and thence the direction of the meridian line, which may be marked off on the earth's surface or plotted in a chart or plan.

The referring mark, which may be any well-defined terrestrial object, should be as far from the observer as possible compatible with distinct vision, and it saves some trouble if a point in the same horizontal plane as the observing station be selected; if this is not practicable, the altitude of the point, if not so small that it may be neglected, must be ascertained, and a corresponding correction applied to the angular distance observed between the sun and the mark.

When the local mean time is accurately known the computation may be carried out in the following manner:

Example 20.—On Sept. 12, 1886, in assumed long. 32° 25′ 45″ E. lat., 34° 46′ 0″ N., bar. 28·7 in., therm. 80°, when the watch, which was 1m. 6s. slow of L.M.T., showed 4h. 42m. 13s., the angular distance, measured with a sextant, between the nearest limb of the sun and a distant point to the right of the sun, was 46° 12′ 40″.

	H. M. S.		° ′ ″	
Time by watch	4 42 13		46 12 40	Instrumental distance sun from point.
Watch slow .	1 6		32	Index correction.
Local M.T.	4 43 19		46 12 8	
E. long. in time	2 9 43	(Table X.)	25	Eccentricity correction.
Approx. G.M.T.	2 33 36		46 11 43	
			15 56	Sun's semi-diam.
			46 27 39	Apparent distance sun's centre from point.

	H. M. S.		° ′ ″	
Local mean time .	4 43 19		4 6 44 N.	Declination (N. A., p. 2).
Equation of time .	3 51		2 23	Variation in 2h. 34m.
Apparent time . .	4 47 10	= 71° 47′ 30″ (hour angle, Table X.)	4 4 21 N.	Corrected declination.

	° ′ ″		
Declination N.	4 4 21	Tan.	8·352470
Hour angle .	71 47 30	Sec.	10·505187
	* 12 50 9	Tan.	9·357657

	° ′ ″				
Hour angle	71 47 30	Tan.	10·482878		
* Lat. 34° 46′ 0″ N. { Diff. if lat. and dec. are same name }	12 50 9	Cosin.	9·989010		
* 12 50 9 { Sum „ „ „ different name † }	21 55 51	Cosec.	10·427724	Cotan.	10·395102
Azimuth of sun . .	82 49 6	Tan.	10·899612	Cosin.	9·096765

		° ′ ″		
True alt. of sun . .		17 14 59 . . .	Tan.	9·492067
(Table I.) Corrected refraction	+	2 49		
		17 17 48		
Parallax (Table V.)	−	8		
App. alt. of sun . .		17 17 40		

† When the hour angle is greater than 6h. or 90° reverse this rule.

	° ′ ″		° ′ ″		
App. dist. sun's centre from point	46 27 39				
App. altitude of sun	17 17 40				
Sum	63 45 19	Half	31 52 39	Tan.	9·793721
Difference	29 9 59	"	14 34 59	"	9·415248
					2) 9·208969
			21 54 43	Tan.	9·604484
			× 2		
Difference of azimuth of sun and point			43 49 26	(point to right of sun).	
Azimuth of sun (as above)			82 49 6		
(Sum if point is right of sun, diff. if to left of it). Azimuth of point			126 38 32	(from south).	
			180		
True bearing of point			306 38 32		

If in the foregoing example the point had had an altitude of say 3° 30′ 0″, the reduced distance 47° 53′ 43″, deduced in the following manner, would have been substituted for the apparent distance, 46° 27′ 39″, in calculating the difference of azimuths of sun and point:—

	° ′ ″		
Altitude of object . .	3 30 0	Sin.	8·785675
Apparent alt. of sun . .	17 17 40	Sin.	9·473168
Apparent distance . .	46 27 39	Cosec.	10·139720
Correction	1 26 4	Sin.	8·398563
Apparent distance . .	46 27 39		
Corrected distance . .	47 53 43		

When the local time is not precisely known, it is necessary to ascertain the sun's apparent altitude by taking observations before and after the time at which the distance is measured, and thence deducing by interpolation the apparent altitude at that instant. The sun's azimuth is then computed in the manner shown in Example 15. For the purpose of reducing the sun's declination an approximate knowledge of the G.M.T. is necessary.

Example 21.—On Nov. 5, 1886, in assumed long. 32° 26′ 0″ E. lat., 34° 46′ 0″ N., the following altitudes of the sun's lower limb and the distance of a distant point from the sun's nearest limb were observed, the point being to the right of the sun. Barom. 30 in., therm. 60°.

Times by Watch.	Double Alt. ☉		Time by Watch.	Distance between Point and Sun's near Limb.	
H. M. S.	° ′ ″		H. M. S.	° ′ ″	
3 39 7	32 33 35		3 42 13 . .	72 33 0	
3 45 40	30 18 50	Watch fast	5 0	20	Index correction.
			3 37 13	72 32 40	
		Long. E. in time	2 9 44 (Table X.)	15	Eccentricity.
		Approx. G.M.T.	1 27 29	72 32 25	
				16 11	Sun's S.D.
		Apparent distance of sun's centre from point		72 48 36	

TO REDUCE THE 2 ALT. ⊙ TO THE WATCH TIME (3H. 42M. 13S.) OF MEASUREMENT OF THE DISTANCE.

	Times by Watch. H. M. S.	2 Alt. ° ′ ″
First obsn. .	3 39 7	32 33 35
Second ,, .	3 45 40	30 18 50
Differences .	6 33	2 14 45

	H. M. S.
Watch time for obsn. of first alt. .	3 39 7
Watch ,, ,, distance .	3 42 13
	3 6

As 6m. 33s. pro. log. (arith. complement)		8·5610
Is to 3m. 6s. ,, ...		1·7639
So is 2° 14′ 45″ ,, ...		·1257
To *decrease* of sun's 2 alt.	° ′ ″ 1 3 47	Pro. log. ·4506
Sun's 1st double alt.	32 33 35	
Sun's 2 alt. at time of obsn. of distance .	31 29 48	

° ′ ″		° ′ ″	
31 29 48	Reduced 2 alt. ⊙	15 45 23 S.	Declination.
20	Index correction.	1 6	Var. 1h. 27m.
31 29 28		15 46 29 S.	Reduced dec.
20	Eccentricity.	90	
2) 31 29 8		105 46 29	Polar distance.
15 44 34	Apparent alt.		
3 20	Refraction (Table I.).		
15 41 14			
8	Parallax (Table V.).		
15 41 22			
16 11	Sun's semi-diam.		

True altitude . .	15 57 33	Sec.	10·017070
Latitude, N. . .	34 46 0	,,	10·085402
Polar distance . .	105 46 29		
Sum	156 30 2		
Half-sum . . .	78 15 1	Cosine	9·308857
P.D.—half-sum . .	27 31 28	,,	9·947832
			2) 9·359161
	28 33 57	Sine	9·679580
	× 2		
Sun's true azimuth .	57 7 54 (from south towards west).		

	° ′ ″			
Apparent distance of sun's centre from point . .	72 48 36			
,, altitude of sun	15 44 34			
Sum . .	88 33 10	Half ° ′ ″ 44 16 35	Tan.	9·989029
Difference	57 4 2	,, 28 32 1	Tan.	9 735372
				2) 9·724401
	36 3 32		Tan.	9·862200
	× 2			
Difference of azimuth of sun and point .	72 7 4 (point to right of sun).			
Azimuth of sun (as above) . . .	57 7 54			
Azimuth of point	129 14 58 (from south).			
	180			
True bearing of point	309 14 58			

A fair approximation to the direction of the meridian line may be obtained, when the sea horizon is available, by measuring the angular distance between a referring mark and the sun at the time of its actual rising or setting, which takes place when its lower limb appears, on account of refraction, to be a little more than its own semi-diameter above the horizon. By applying the distance to the computed amplitude of the sun the true bearing of the referring mark is deduced.

Since, by reason of the uncertainty of the value of the refraction of objects observed within a degree or two of the horizon, we cannot tell by observation the exact instant of actual sunrise or sunset, this method cannot be relied upon to give perfectly accurate results, but in most cases it is sufficiently exact for the purpose of determining the error of the compass.

Example 22.—On Nov. 4, 1886, in assumed long. 32° 30′ 0″ E., lat. 34° 46′ 0″ N., the angular distance between a terrestrial mark and the sun's near limb was found to be 57° 46′ 40″ at about 5h. 10m. local mean time, when the sun's lower limb was about 20′ above the horizon. Mark to the right of the sun.

	H. M.		° ′ ″		° ′ ″
Local mean time	5 10	Sun's declination	15 27 3·5 S.	Distance sun's near limb and mark	57 46 40
E. long. in time	2 10	Variation in 3h.	2 18·4	Sun's semi-diam.	16 10
Approx. G.M.T.	3 0	Reduced declination	15 29 22 S.	Dist. sun's centre and mark	58 2 50

	° ′ ″		
Declination .	15 29 22	Sin.	9·426613
Latitude .	34 46 0	Sec.	10·085402
Sun's amplitude (south of west) . .	18 58 17	Sin.	9·512015
Bearing of west	270		
Sun's true bearing	251 1 43		
Distance sun's centre from mark . .	58 2·50 (mark to the right of the sun).		
True bearing of mark	309 4 33		

OBSERVATIONS OF PLANETS.

When a planet is employed in place of a fixed star in any of the foregoing observations, corrections for the planet's parallax in altitude (see Table VI.) must be added in deducing the true from the observed altitude, and if, as is sometimes the case, the planet's limb, instead of its centre, be observed (see page 30), the value of its semi-diameter must be applied.

In determining latitude by a planet's meridian altitude the declination "at transit at Greenwich," taken from the Nautical Almanac, and corrected for the longitude of the place of observation by means of the "variation of dec. in 1 hour of longitude," may be employed; or, if preferred, the declination registered under the name of the planet at "Mean Time" may be taken and reduced to the instant of observation, the Greenwich date of which can be found by applying the longitude in time to the mean time of the meridian passage at Greenwich, corrected for the longitude of the place of observation, if it differs much with that of Greenwich.

Full instructions for reducing the declination, right ascension, and time of meridian passage, to any time and longitude, are given in the Nautical Almanac in the explanation of the Planetary Ephemerides at mean noon, and at transit.

Example 23.—On April 11, 1866, in long. 32° 30′ E the double meridian altitude of *Jupiter's* centre was observed to be 114° 34′ 35″; bar. 29·25 in., therm. 50°. Planet south of observer.

At Transit at Greenwich" (Almanac).

″	
6·3	Var. of dec. in 1 hour of long.
×2·17	E. long. (2h. 10m. = 2·17h.).
13·6	Reduction.

° ′ ″	
2 2 22·5	N. Declination (increasing).
− 13·6	Reduction for east long.
2 2 8·9	Reduced declination, N.

or

"Mean Time". (Almanac).

H. M.	
10 36·7	G.M.T. of mer. passage at Greenwich.
·4	Cor. (var. 4·3m. daily, & long. 2h. 10m.).
10 37·1	Local mean time of transit at place.
2 10·0	E. long. in time (Table X.).
8 27·1	G.M.T. of transit at place.

° ′ ″	
2 1 15·3	N. dec. 11 April.
2 3 46·5	N. „ 12 „
2 31·2	Increase of dec. in 24 hours.

As 24 H. : 8 H. 27·1 M. :: 2′ 31·2″ : 53·2″.

° ′ ″	
2 1 15·3	N. dec. (increasing) 11 April.
53·2	Increase in 8h. 27·1m. (Greenwich date).
2 2 8·5	Reduced declination, N.

° ′ ″		
114 34 35		Instrumental 2 alt. of *Jupiter's* centre.
25	−	Index correction.
114 34 10		
10		Eccentricity correction.
2)114 34 0		
57 17 0		Obsd. alt.
37	−	Refraction (Table I.).
57 16 23		
1	+	Parallax in alt. (hor. par. per Almanac 2″). Table VI.
57 16 24		True alt.
90		
32 43 36		N. mer. zenith distance.
2 2 9		N. declination.
34 45 45		N. latitude.

In determining time by a planet's altitude the right ascension and declination must be taken from the page of the Nautical Almanac containing the planet's elements at "mean time," and reduced to the instant of observation, an approximate Greenwich date being deduced from the assumed local mean time and longitude.

The planet's horizontal parallax and semi-diameter are given in the Almanac, under the planet's name, "at transit at Greenwich."

Example 24.—On December 20, 1866, in long. 32° 25′ 45″ E., lat. 34° 46′ 0″ N., the double altitude of *Saturn's* centre was observed to be 93° 3′ 15″ when the watch showed 10h. 21m. 27·5s. Planet east of the meridian. Bar. 30 in., therm. 60°.

Declination.

°	′	″	
21	39	14·2	N. on 20th Dec.
21	39	59·9	N. on 21st „
		45·7	Increase of decl. in 24 hours.

H.	M.	S.	
10	21	27·5	Time by watch of obsn.
2	9	43	E. long. in time (Table X.).
8	11	44·5	Approx. Greenwich date.

As 24 H. : 8 H. 11·7 M. :: 45·7″ : 15·6″

°	′	″	
21	39	14·2	N. dec. (increasing) 20th Dec.
		15·6	Increase in 8h. 11·7m. (Greenwich date).
21	39	30	Reduced declination.

H.	M.	S.	
17	55	58·40	Sidereal time at noon
	1	20·78	Acceleration for 8h. 11m. 44s. (Table IX.)
17	57	19·2	R.A. of mean sun.

Saturn's Right Ascension.

H.	M.	S.	
7	29	11·98	on 20th Dec.
7	28	53·27	on 21st „
		18·71	Decrease of R.A. in 24 hours.

As 24 H. : 8 H. 11·7 M. :: 18·7 S. : 6·4 S.

H.	M.	S.	
7	29	11·9	R.A. on 20th Dec.
		6·4	Decrease in 8h. 11·7m. (Greenwich date).
7	29	5·5	Reduced R.A.

°	′	″	
93	3	15	2 alt. of *Saturn's* centre.
		5	Index correction.
93	3	10	
		10	Eccentricity correction.
2)93	3	0	
46	31	30	Observed alt.
		54	Refraction (Table I.).
46	30	36	
		1	Parallax (hor. par. 1″·1 per N.A.) Table VI.
46	30	37	True alt.
90			
43	29	23	Zenith distance.

Latitude	34 46 0 N.		Sec.	10·085402
Declination	21 39 30 N.		Sec.	10·031797
Difference (same names)	13 6 30			
Zenith distance	43 29 23			
Diff. + Z.D.	56 35 53	Half = 28 17 56	Sin.	9·675843
,, ,,	30 22 53	,, = 15 11 26	Sin.	9·418351
			2)	9·211393
Half-hour angle	23° 47′ 18″		Sin.	9·605696
	× 2			
	47 34 36 = (in time)	H. M. S. 3 10 18·4 (Table X.).		

	H. M. S.	
R.A. of *Saturn*	7 29 5·5	
Hour angle (east of meridian −)	3 10 18·4	
R.A. of the meridian	4 18 47·1	
Added to enable R.A. of mean sun to be subtracted	24	
	28 18 47·1	
R.A. of the mean sun	17 57 19·2	
	10 21 27·9	Local mean time.
	10 21 27·5	Time by watch.
	0 0 0·4	Watch slow.

OBSERVATIONS OF THE MOON.

On account of the rapidity in the change in the Moon's declination and right ascension it is not advisable to determine latitude or time by observations of the Moon's altitude.

The method of deducing the true altitude of the Moon from an observation will be found in Example 14. The Moon's parallax in altitude is found by adding together the log. secant of the apparent altitude (rejecting 10 in the index) and the proportional logarithm of the Moon's reduced horizontal parallax, the sum being the proportional log. of the parallax in altitude. By subtracting from this the refraction corresponding to the apparent altitude, the "Moon's correction," which is often given in Nautical Tables, is obtained.

COMBINATION OF THE RESULTS OF OBSERVATIONS.

From what has already been stated it is evident that any determination of latitude, longitude, time, or the direction of the meridian by means of a single observation, can only be regarded as an approximation, near or otherwise according to the amount of care which has been bestowed upon the observation, and the conditions under which it has been made. To obtain a value approaching more closely to absolute truth it is necessary to use the mean of deductions from several observations taken under conditions tending to balance the unavoidable errors of one observation by those of another; to this end each observation should be taken with the utmost care, and that having been done, its deduction should be recorded, however much it may differ with previous determinations, and it should not be excluded from the discussion of the value sought until it has been proved, by comparison with a considerable number of other values, to be abnormally erroneous. To enable the reliability or weight of each observation to be duly considered a note should be made of the conditions under which it was taken (whether the horizon mercury was covered or uncovered, steady or tremulous, which telescope and which shades were employed, whether the observer judged the observation as far as he personally was concerned, to be *excellent*, *good*, *indifferent*, or *bad*, &c.), and these particulars should be recorded at the time that the instrumental readings are written down and *before* any computations are made, thus preventing the judgment being biassed by any existing knowledge of the quantity it is desired to determine. If this precaution be neglected there is a liability to unduly assign to error of observation any discrepancy between the value deduced and that previously

found, and under these circumstances it is probable that less weight will be given to the observation than is due to it, and than it would have received had the observer been without expectation as to the result.

IDENTIFICATION OF STARS.

In order *readily* to recognize stars in the heavens, it is necessary in the first instance to be acquainted with a few of the more conspicuous constellations, such as the Great Bear (called also the *Plough*, *Charles' Wain*, or the *Dipper*), and Cassiopeia, in the northern skies; Orion and the Eagle, near the equinoctial; and the Southern Cross, the Southern Fish, and the Scorpion, in the southern skies. When these are known, single stars can be traced with the aid of a star map, or by descriptions of their positions with respect to one or other of the constellations. Thus, by turning towards the Great Bear and drawing an imaginary line from β through α (the Pointers), and producing it about five times the distance between these stars, the line will terminate near the Pole Star, which may then be identified; again, by tracing through the Pole Star a line at right angles to that already drawn, it will be found to pass close to the bright stars Vega, on one side of the Pole, and Capella, on the other; the star nearest the tail of the Great Bear being Vega. In this way, with very little study, the observer may become acquainted with the positions of all the stars he is likely to use in sextant observations.

Should he, however, be quite unacquainted with any of the constellations, he will still be able to employ the stars for determining latitude, as when any bright star which it may be desired to observe is on or near the meridian, it can easily be identified in the following manner:—The approximate time of the star's transit having been computed by subtracting from its right

ascension (increased if necessary by 24 hours) the sidereal time at mean noon (Nautical Almanac, page II.), a look-out is *then* kept for the star's appearance in the direction of the meridian, which latter, if unknown, may be determined with sufficient accuracy by the compass. The meridian altitude having been roughly found by the formula—

$$\text{Alt.}\left\{\begin{array}{l}\text{Reckoned from the south in}\\ \text{north latitudes, and from}\\ \text{the north in south lati-}\\ \text{tudes}\end{array}\right\}\begin{array}{l}=\text{Co. lat.} + \text{dec. (when the lat. and dec. are of the same name)}\\ =\text{Co. lat.} \backsim \text{dec. (when the lat. and dec. are of contrary names)}\end{array}$$

the corresponding angular distance above the north or south point of the horizon can generally be estimated accurately enough by eye to enable the star, whose magnitude will of course have been ascertained, to be identified as it approaches the meridian. If any doubt exists as to the star, it may be removed by testing its altitude with the sextant, the index being clamped at the computed meridian altitude.

For example, suppose on January 1, 1887, in latitude 35° north, it was required to find the star *a* Canis Majoris (*Sirius*):

	H. M.		° ′
Star's R.A.	6 40	Latitude	35 0 North.
Add, to permit of S.T. being subtracted .	+ 24		90
	30 40	Co. lat.	55 0
Sidereal time at noon .	18 43	Star's declination . .	16 34 South.
Approx. mean time of transit . .	11 57	,, merid. alt. (approx.)	38 26 *

Now, by looking a little before midnight towards the south point of the horizon, and estimating therefrom a vertical arc of 38°, the observer could hardly fail to discover the star *Sirius*. In like manner, any other first or second magnitude star can be identified.

The lunar distance stars and planets (see page 60) can be found without difficulty, provided the moon be visible,

* Difference, lat. and dec. being of contrary names.

by means of their distances east or west from that body. Extract from the Nautical Almanac the star's lunar distance corresponding to a rough estimate, which must be made, of the Greenwich mean time, and note whether the star is east or west of the moon. Clamp the index of the sextant at the given angle, and, without inserting the telescope, so hold the sextant that, by looking through the ring or collar, the image of the moon may be seen *reflected* from the silvered part of the horizon glass, which must be directed towards that portion of the sky lying to the east or west of the moon according to the direction of the star sought.* While keeping the moon's reflected image in view, give the sextant a slow rotary motion, looking at the same time through the transparent part of the horizon glass until, by thus sweeping with the eye an arc of the heavens, the star be discovered; it will be found that the rotation of the sextant will bring the reflected image of the moon either directly in contact with or very near to the star. When the angular distance is not very great, no difficulty will be experienced in keeping the reflection of the moon constantly visible, but with large angles some dexterity is required to catch the moon's reflection in the horizon glass, and to keep it there when the sextant is moved. The observation is facilitated by resting the exterior of the telescope collar just below the observer's eye and on his cheek, maintaining the contact, while the instrument is rotated, by a corresponding movement of the head. Both eyes should be kept open. In cases where the proper position of the sextant for bringing the moon's reflection into sight cannot be ascertained, the observer may first hold the index at zero, and, looking directly at the moon through the transparent part of the horizon glass, gradually move the index forward, keeping, as the images separate, the reflected one in sight in the silvered part of the glass until he estimates that the index is at about the required angle; by then care-

* A shade should be turned up before the *index* glass.

fully noting the position of the plane of the sextant and the direction, with respect to the sky, of the line of sight passing through the transparent portion of the horizon glass; he will probably be able, after the index has been more accurately set at the computed lunar distance, to replace the sextant in the position necessary for making the observation.

DEFINITIONS.

Acceleration.—An additive correction applied to an interval of mean solar time to convert it into sidereal time. The acceleration for 24 hours' solar time is 3m. 56·555s.; therefore 24 hours, or 1 solar day, = 24h. 3m. 56·555s. sidereal time (see Table IX.).

Altitude.—The arc of a circle, perpendicular to the horizon, intercepted between the horizon and some object above it, the observer's eye being at the centre of the circle.

It is the complement of the zenith distance—

$$\text{Alt.} = 90° - \text{zenith dist.}$$

Instrumental altitude is the reading of the instrument when an altitude has been observed with it.

Apparent or *observed altitude* is the instrumental altitude freed by the application of proper corrections, from index and eccentricity errors.

The *true altitude* of a celestial object is deduced from the apparent altitude by adding the parallax, and subtracting the refraction and also the *dip*, when the sea horizon is used. With the sun, moon, and occasionally with planets, the true altitude of the centre is found by applying the value of the semi-diameter to the true altitude of the limb.

Amplitude.—The angular distance of a heavenly body at the time of its rising or setting, from the east or west point of the horizon:

$$\text{Sin. amplitude} = \text{sin. decln.} \times \text{sec. lat.}$$

Apparent time.—An apparent or solar day is the interval

between two successive upper transits of the sun across a given meridian; it is a variable quantity, and is divided primarily into 24 hours. Any portion of this day is called apparent time which can be converted into mean time by applying a correction known as the *equation of time*. Apparent time is the hour angle of the true sun.

Augmentation of the Moon's Semi-diameter.—An additive correction applied to the registered semi-diameter of the moon, which is the angle subtended by the radius of its disc at the centre of the earth. Since, to an observer on the earth's surface, the moon is nearer by a semi-diameter of the earth when she is in the zenith than when in the horizon, her apparent size varies with the altitude, and this necessitates the correction of the value of the semi-diameter given in the Almanac (see Table VII.). The augmentation of the sun's semi-diameter is insensible, owing to its great distance from the earth.

Azimuth.—The azimuth of a heavenly body is an arc of the horizon intercepted between the north or south point and the point where a vertical circle passing through the body cuts the horizon. In north latitudes the azimuth is measured from the south point, and in south latitudes from the north.

Bearing.—The horizontal angular distance between an object and the north point. It is reckoned from zero (north) round to the right up to 360°. The bearing is true or magnetic according as it is reckoned from the terrestrial or from the magnetic pole.

Civil Day.—Twenty-four hours divided into two equal parts, distinguished as *ante meridiem* and *post meridiem*. It commences at midnight—*i.e.*, 12 hours before the astronomical day of the same date. To convert civil into astronomical reckoning, add 12 hours to the civil A.M. time, and prefix the date of the previous day, thus:

9h. 30m. A.M. on 8th Nov. civil reckoning = 7th Nov. 21h. 30m. astronomical reckoning.

If the civil time is P.M. its reckoning is entirely the same as the astronomical time, thus:

5 P.M. on 8th Nov. civil time = 8th Nov. 5h. astronomical time.

Day.—The interval between two successive upper transits of a celestial body across a given meridian. It is called a solar day or a sidereal day according as the sun or a star is the body to which it is referred. The astronomical day commences at noon, while the civil day commences at the previous midnight, as it was deemed more convenient in ordinary life to begin the day when the sun crosses the meridian at its *lower* transit.

Declination.—An arc of a meridian contained between the centre of an object and the equinoctial. It is measured north or south from the equinoctial up to 90°.

Dip of the Horizon.—The angle at the eye of the observer contained by the planes of the sensible and visible horizons. The quantity depends on the height of the eye above the level of the sea; the correction for dip to be applied to an observed altitude is always subtractive (see Table IV.).

Double Altitude (2 alt.).—The instrumental reading when an altitude has been observed by means of an artificial horizon. To obtain the apparent altitude this reading must be divided by 2 *after* the necessary corrections for instrumental errors have been applied.

Ecliptic.—The great circle in the heavens which the sun, in consequence of the earth's motion in its orbit, appears to describe in the course of a year. It is divided into twelve equal parts, called *signs*, and is inclined to the equinoctial; the two points where it cuts it are called the equinoxes, or equinoctial points.

Equation of Time.—The difference between apparent and mean time; or, it is the angular distance in time between the *true* and the *mean suns.*

In the Nautical Almanac its value at both apparent and mean noon at Greenwich is given for each day of the year, and it serves, when reduced to the instant of observation, to convert apparent into mean time, and *vice versâ.*

Equinoctial or Celestial Equator.—The plane of the terrestrial equator produced to the heavens.

First Point of Aries (♈).—The point in the ecliptic where the sun ascends from the southern to the northern hemisphere at the vernal equinox. Owing to the precession of the equinoxes this point is no longer in the constellation *Aries,* but it still retains the name given to it by the ancient astronomers. It is the zero point whence right ascensions are measured.

Greenwich Date.—The astronomically expressed day of the month and time at the meridian of Greenwich at any absolute instant. In nearly all astronomical computations an approximate knowledge of the Greenwich date is necessary to enable the elements given in the Nautical Almanac to be reduced to the instant of observation.

$$\text{Greenwich date} \left\{ \begin{array}{l} = \text{Local mean time} + \text{west longitude (expressed in time).} \\ = \text{Local mean time} - \text{east longitude (\quad ,, \quad\quad ,, \quad).} \end{array} \right.$$

Horizon.—The *visible* or *apparent horizon* is the circle formed by the apparent meeting of the sea and the sky, and limiting the range of vision; which latter, however, varies with the height of the observer's eye above the level of the sea.

The *sensible horizon* is a circle whose plane passes through the eye of the observer, and is perpendicular to a straight line extending from his zenith to the centre of the earth.

The *rational horizon* is the circle whose plane is

parallel to the sensible horizon, and, passing through the centre of the earth, is produced to the heavens.

An *artificial horizon* is an instrument affording a fluid or solid horizontal plane with a reflecting surface whence the rays of light from an object can be thrown back to the eye of an observer suitably placed to receive them. The angle measured by means of an artificial horizon is double the altitude of the object above the *sensible* horizon.

Horizontal Parallax. — The horizontal parallax of a heavenly body is the angle under which the earth's equatorial semi-diameter would appear at the centre of the body. The horizontal parallax of the moon, owing to her proximity to the earth, requires reduction for the latitude of the observer. (See Table VIII.)

Hour Angle.—The hour angle of a heavenly body is the angle at the pole between the meridian of the place of observation and the *hour circle* or *circle of declination* passing through the body; it is measured by the arc of the equinoctial between the meridian and the hour circle. The hour angle determines the meridian distance, or the interval between the instant of observation and the instant of the body's transit across the observer's meridian. It is generally reckoned from the meridian westward through 24 hours, or 360°; but for the purposes of calculation it may also be reckoned from the meridian towards the east: thus the hour angle 15 hours (or 225°) = 9 hours (or 135°) *east.*

Hour Circles, or *Circles of Declination,* are great circles passing through the poles of the celestial sphere. They correspond to meridians on the terrestrial sphere.

Latitude is the angle at the earth's centre measured by the arc of the meridian between the equator and the place. Astronomical latitude is the angle contained between the plane of the equator and the plumb-line

at the place. It is equal to the altitude of the celestial pole, to the declination of the zenith, and to the zenith distance of the meridian point of the equator. It is reckoned from the equator (0°) north and south up to 90°, which is the latitude of the poles.

Latitude	= South zenith distance ∾ north declin.		Of a celestial body.
	= South " + south "		
	= North " + north "		
	= North " ∾ south "		

Limb.—The edge of the disc of the sun, moon, or of a planet. The following symbols are commonly used.

Upper limb of sun	$\overline{\odot}$	Upper limb of moon	$\overline{☽}$
Lower " "	$\underline{\odot}$	Lower " "	$\underline{☽}$

Nearest limbs of moon and sun ☽| |⊙
Near limb of moon and far limb of sun ☽| ⊙|

Local time.—The time at the meridian of the place of observation. The difference between the local times of any two places is equal to the difference of their longitudes in time (1 hour = 15°).

Longitude.—An arc of the equator between the meridian of the place and the "first meridian," or it is the angle at the pole included between the meridian of the place and any "first meridian," which is generally assumed to be that of Greenwich Observatory, in longitude 0° 0′ 0″. Longitude is reckoned east and west from the "first meridian" up to 180°, or 12 hours; it may be expressed in degrees, &c., of arc, or in time. (See Table X.)

Mean sun.—See Mean time.

Mean time.—The time denoted by an imaginary sun, called the *mean sun*, which is supposed to move uniformly in the equator. The interval between the departure of a meridian from the *mean sun* and its return to it is the mean solar day, which is not a variable quantity, as is the apparent day. Clocks and watches are adjusted with a view to their keeping

mean time, and it is by their indications that we ascertain the position of the fictitious sun. Mean time is deduced from apparent time, and *vice versâ*, with the aid of the *equation of time* which is given in the Nautical Almanac. The equation of time must be taken from page I. in reducing apparent to mean time, and from page II. when converting mean into apparent time.

Meridian.—A celestial meridian is the terrestrial meridian of the place extended to the heavens, and is a great circle passing through the poles and the zenith of the place of observation. On this circle the latitude of the place is reckoned.

The *first meridian* is the meridian from which terrestrial longitude is reckoned, different nations recognizing different *first meridians.* The meridian of Greenwich Observatory is the one chosen by most maritime nations.

Meridian Line.—A straight line passing through the true north and south points of the horizon.

Nadir.—The point in the celestial concave below the horizon and directly opposite the zenith (see *Zenith*).

Noon.—The instant when the centre of the sun crosses the meridian of the place above the horizon. It is apparent noon or mean noon according as the real sun or the mean (imaginary) sun is the body referred to.

Parallax in Alt.—The difference between the apparent situations of a celestial body when viewed respectively from the surface and from the centre of the earth. The parallax in alt. of a body is greatest when it is in the horizon; when it is in the zenith parallax vanishes. In deducing the true place of a heavenly body from the apparent place the correction for parallax is always additive. Owing to the immense distance from the earth of the fixed stars, their parallax is insensible. (See Tables V. and VI.)

Sin. parallax = sin. horizontal parallax × cos. altitude.

Planet.—A heavenly body belonging to our solar system, and revolving round the sun in an elliptic orbit.

Polar Distance.—The arc of the meridian contained between either pole of the equinoctial and the centre of the object.

In north latitudes, polar distance = 90° − N. decl. = 90° + S. decl.
In south latitudes, polar distance = 90° + N. decl. = 90° − S. decl.

Poles.—The poles of the celestial sphere are the extremities of its axis, and are fixed points about which the heavenly bodies appear to revolve. The poles of the earth are the extremities of its axis, and are equidistant from every part of the equator. The altitude of the celestial pole above the horizon is equal to the latitude of the place.

Prime Vertical.—A great circle passing through the zenith and east and west points of the horizon.

Refraction.—The apparent displacement of an object caused by the bending of the rays of light proceeding from it in passing through the atmosphere. Its effect is greatest when the object is in the horizon. Refraction causes objects to appear more elevated than they really are; so a subtractive correction must be applied to the apparent altitude to free it from refraction. (See Tables I. II. and III.)

Retardation.—A subtractive correction applied to an interval of sidereal time to convert it into mean solar time. The retardation for 24 sidereal hours is 3m. 55·909s.; therefore, 24 sid. hours, or 1 sid. day, =23h. 56m. 4·090s. solar time.

Right Ascension.—The R.A. of a celestial body is the arc of the equator intercepted between the "first point of Aries" and the circle of declination, or hour circle, passing through the body. It is measured from west to east through 360° or 24 hours.

The R.A. of a body is the *local* sidereal time of its crossing the meridian of the place of observation.

Right Ascension of the Mean Sun.—The arc of the equator intercepted between the "first point of Aries" and the hour circle passing through the fictitious or mean sun.

To determine the R.A. of the mean ☉ accelerate (Table IX.) for the *Greenwich date* the sidereal time at Greenwich noon registered in page II. of the Nautical Almanac; the result is the R.A. of the mean ☉.

Right Ascension of the Meridian.—The sidereal time at the place of observation. It can be found by adding the local mean time to the R.A. of the mean ☉ (which see).

Semi-Diameter.—The semi-diameter of a heavenly body is the angle, at the place of observation, subtended by the radius of its disc.

Sidereal Time.—The hour angle of the "first point of Aries" reckoned westward from the meridian of the place of observation. The sidereal day is the interval between two successive upper transits of the "first point of Aries" across the meridian. Sidereal days are *sensibly* equal. The sidereal time at Greenwich mean noon for each day of the year is registered in page II. of the Nautical Almanac.

Star.—A self-luminous celestial body, apparently preserving the same situation with respect to other stars. The changes in a star's declination and right ascension are practically insensible for short periods of time, and it is chiefly on this account that stellar observations are preferable to those of the sun, moon, or planets, which rapidly change their places in the heavens.

Sun ☉.—The ruling orb of our solar system. To other stars our sun is a fixed star.

Transit (*Meridian Passage* or *Culmination*).—The passage of the centre of a heavenly body across the meridian of the observer. The passage above the observer's

horizon is termed the *upper* transit; the other passage is the *lower* transit.

Vertical Circle.—A great circle passing through the zenith and nadir, and intersecting the horizon at right angles.

Vertical Line.—A straight line, or any portion of it, from the zenith to the centre of the earth.

Zenith.—The point in the celestial concave farthest from the observer's horizon and directly above his head. It is the point directly opposite the nadir. The altitude of the zenith is 90°.

Zenith Distance.—The angular distance from the zenith measured on a vertical circle. It is the complement of the altitude.

$$\text{Z.D.} = 90° - \text{alt.}$$

When an object is on the meridian, its zenith distance is called the *meridian zenith distance*, and it is named north or south according as the zenith is north or south of the body.

Table I.—*Mean Astronomical Refraction.*

(Barometer, 30 inches. Fahrenheit's Thermometer, 50°.)

Correction to be subtracted from the apparent altitude.

App. Alt.	Refrac.	App. Alt.	Refrac.	App. Alt.	Refrac.	App. Alt.	Refrac.	App. Alt.	Refrac.
° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″	° ′	′ ″
5 0	9 52	8 20	6 19	13 40	3 55	27 0	1 54	54 0	0 42·5
5 5	9 44	8 25	6 15·5	13 50	3 52·5	27 30	1 51·5	55 0	0 41
5 10	9 36	8 30	6 12	14 0	3 49·5	28 0	1 49·	56 0	0 39·5
5 15	9 28·5	8 35	6 8·5	14 10	3 47	28 30	1 47·	57 0	0 38
5 20	9 21	8 40	6 5	14 20	3 44	29 0	1 45·	58 0	0 36·5
5 25	9 14	8 45	6 2	14 30	3 41·5	29 30	1 42·5	59 0	0 35
5 30	9 7	8 50	5 59	14 40	3 39	30 0	1 40·5	60 0	0 33·5
5 35	9 0	8 55	5 55·5	14 50	3 36·5	30 30	1 38·5	61 0	0 32·5
5 40	8 53·5	9 0	5 52·5	15 0	3 34	31 0	1 36·5	62 0	0 31
5 45	8 47	9 5	5 49·5	15 10	3 31·5	31 30	1 34·5	63 0	0 30
5 50	8 40·5	9 10	5 46·5	15 20	3 29·5	32 0	1 33	64 0	0 28·5
5 55	8 34	9 15	5 43·5	15 30	3 27	32 30	1 31	65 0	0 27
6 0	8 28	9 20	5 40·5	15 40	3 25	33 0	1 29·5	66 0	0 26
6 5	8 22	9 25	5 38	15 50	3 22·5	33 30	1 28	67 0	0 24·5
6 10	8 16	9 30	5 35	16 0	3 20·5	34 0	1 26	68 0	0 23·5
6 15	8 10·5	9 35	5 32·5	16 10	3 18·5	34 30	1 24·5	69 0	0 22·5
6 20	8 5	9 40	5 29·5	16 20	3 16·5	35 0	1 23	70 0	0 21
6 25	7 59·5	9 50	5 24·5	16 30	3 14	35 30	1 21·5	71 0	0 20
6 30	7 54	10 0	5 19	16 40	3 12	36 0	1 20	72 0	0 19
6 35	7 48·5	10 10	5 14	16 50	3 10·5	36 30	1 18·5	73 0	0 18
6 40	7 43·5	10 20	5 9·5	17 0	3 8·5	37 0	1 17	74 0	0 16·5
6 45	7 38·5	10 30	5 4·5	17 30	3 3	37 30	1 16	75 0	0 15·5
6 50	7 33·5	10 40	5 0	18 0	2 57·5	38 0	1 14·5	76 0	0 14·5
6 55	7 28·5	10 50	4 55·5	18 30	2 52·5	38 30	1 13	77 0	0 13·5
7 0	7 24	11 0	4 51	19 0	2 48	39 0	1 12	78 0	0 12·5
7 5	7 19	11 10	4 47	19 30	2 43	39 30	1 10·5	79 0	0 11·5
7 10	7 14·5	11 20	4 43	20 0	2 39	40 0	1 9·5	80 0	0 10·5
7 15	7 10	11 30	4 39	20 30	2 34·5	41 0	1 7	81 0	0 9
7 20	7 5·5	11 40	4 35	21 0	2 30·5	42 0	1 4·5	82 0	0 8
7 25	7 1·5	11 50	4 31	21 30	2 27	43 0	1 2·5	83 0	0 7
7 30	6 57	12 0	4 27·5	22 0	2 23·5	44 0	1 0·5	84 0	0 6
7 35	6 53	12 10	4 24	22 30	2 20	45 0	58	85 0	0 5
7 40	6 49	12 20	4 20·5	23 0	2 16·5	46 0	56	86 0	0 4
7 45	6 45	12 30	4 17	23 30	2 13·5	47 0	54·5	87 0	0 3
7 50	6 41	12 40	4 13·5	24 0	2 10	48 0	52·5	88 0	0 2
7 55	6 37	12 50	4 10·5	24 30	2 7	49 0	50·5	89 0	0 1
8 0	6 33·5	13 0	4 7	25 0	2 4·5	50 0	49	90 0	0 0
8 5	6 29·5	13 10	4 4	25 30	2 1·5	51 0	47	...	...
8 10	6 26	13 20	4 1	26 0	2 0	52 0	45·5	...	...
8 15	6 22·5	13 30	3 58	26 30	1 56·5	53 0	44	...	...

See Example at foot of Table II.

Table II.—*Correction of the Mean Refraction for the Height of Fahrenheit's Thermometer.*

Correction to be applied to the mean refraction extracted from Table I.

App. Alt.	Height of the Thermometer.														
	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
	Add	Add	Add	Add	Add	Add		Subt.	Subt.	Subt.	Subt.	Subt.	Subt.	Subt.	Subt.
°	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
5	42	35	28	20	13	7	0	6	13	19	25	31	37	42	48
6	35	29	23	17	11	6	0	5	11	16	21	26	31	36	40
7	30	25	20	15	10	5	0	5	9	14	18	22	27	31	35
8	26	21	17	13	8	4	0	4	8	12	16	19	23	26	30
9	24	19	15	11	8	4	0	4	7	11	14	18	21	24	27
10	21	17	14	10	7	3	0	3	7	10	12	16	19	22	25
11	18	16	13	9	6	3	0	3	6	9	12	14	17	20	22
12	17	14	11	8	6	3	0	3	5	8	10	13	15	18	20
13	15	13	10	7	5	2	0	2	5	7	9	11	14	16	18
14	14	12	10	7	5	2	0	2	5	7	9	11	14	16	18
15	13	11	9	6	4	2	0	2	4	6	8	10	12	14	16
16	12	10	8	6	4	2	0	2	4	6	8	10	12	14	15
17	11	9	7	6	4	2	0	2	3	5	7	8	10	12	13
18	11	9	7	6	4	2	0	2	3	5	7	8	10	12	13
19	10	9	7	6	4	2	0	2	3	5	7	8	10	12	12
20	9	8	6	5	3	1	0	1	3	4	6	7	8	10	11
25	8	6	5	4	2	1	0	1	2	3	5	6	7	8	9
30	7	5	4	3	2	1	0	1	2	3	3	4	5	6	7
40	4	3	2	2	1	1	0	1	1	2	2	3	3	4	4
50	3	3	2	2	1	1	0	1	1	2	2	2	3	3	3
60	2	2	1	1	1	0	0	0	1	1	1	1	2	2	2
70	2	1	1	1	1	0	0	0	0	0	0	0	1	1	1

Example.—Required the refraction corresponding to alt. 27°. Barom. 28·7 in., therm. 70°.

			′	″
By Table I.	Alt. 27° Mean refraction . .		1	54
,, ,, II.	Correction for therm. 70° . .	—		4·2
			1	49·8
,, ,, III.	,, ,, barom. 28·7 in. .	—		4·6
	Reduced refraction		1	45·2

TABLE III.—*Correction of the Mean Refraction for the Height of the Barometer.*

Correction to be applied to the mean refraction extracted from Table I.

App. Alt.	HEIGHT OF THE BAROMETER.—Inches.													
	28·00	28·25	28·50	28·75	29·00	29·25	29·50	29·75	30·00	30·25	30·50	30·75	31·00	31·25
	Subt.	Subt.	Subt.	Subt.	Subt.	Subt.	Subt.	Subt.		Add	Add	Add	Add	Add
°	″	″	″	″	″	″	″	″	″	″	″	″	″	″
5	41	35	30	25	20	15	10	5	0	5	10	15	20	25
6	34	30	26	21	17	13	9	4	0	4	9	13	17	21
7	30	26	23	19	15	11	8	4	0	4	8	11	15	19
8	26	23	20	16	13	10	7	3	0	3	7	10	13	16
9	24	21	18	15	12	9	6	3	0	3	6	9	12	15
10	21	19	16	14	11	8	6	3	0	3	6	8	11	14
11	20	18	15	13	10	8	5	3	0	3	5	8	10	13
12	18	16	14	11	9	7	5	2	0	2	5	7	9	11
13	18	16	14	11	9	7	5	2	0	2	5	7	9	11
14	15	13	11	10	8	6	4	2	0	2	4	6	8	10
15	14	12	10	9	7	5	3	2	0	2	3	5	7	9
16	14	12	10	9	7	5	3	2	0	2	3	5	7	9
17	12	10	9	7	6	4	3	1	0	1	3	4	6	7
18	12	10	9	7	6	4	3	1	0	1	3	4	6	7
19	11	10	9	7	6	4	3	1	0	1	3	4	6	7
20	10	9	7	6	5	4	2	1	0	1	2	4	5	6
25	8	7	6	5	4	3	2	1	0	1	2	3	4	5
30	6	5	4	4	3	2	1	1	0	1	1	2	3	4
40	4	3	3	2	2	1	1	0	0	0	1	1	2	2
50	3	3	3	2	2	1	1	0	0	0	1	1	2	2
60	2	2	1	1	1	1	0	0	0	0	0	1	1	1
70	2	1	1	1	1	0	0	0	0	0	0	0	1	1

See Example at foot of Table II.

Table IV.—*Dip of the Horizon.*

Correction to be subtracted from the apparent altitude.

Height of the Eye.	Dip of the Horizon.	Height of the Eye.	Dip of the Horizon.
Feet.	′ ″	Feet.	′ ″
1	1 0	22	4 34
2	1 22	24	4 48
3	1 41	26	5 0
4	1 59	28	5 11
5	2 11	30	5 22
6	2 22	35	5 49
7	2 38	40	6 11
8	2 48	45	6 36
9	2 58	50	6 58
10	3 8	55	7 18
11	3 13	60	7 38
12	3 22	65	7 56
13	3 32	70	8 11
14	3 40	75	8 30
15	3 49	80	8 48
16	3 58	85	9 0
17	4 2	90	9 19
18	4 10	95	9 33
19	4 16	100	9 49
20	4 22		

Table V.—*Sun's Parallax in Altitude.*

Correction to be added to the apparent altitude.

App. Alt.	Parallax.	App. Alt.	Parallax.
°	″	°	″
10	8·5	50	5·5
15	8·5	55	5
20	8	60	4
25	8	65	3·5
30	7·5	70	3
35	7	75	2·5
40	6.5	80	1·5
45	6	85	0·5

Table VI.—*Planet's Parallax in Altitude.*

Correction to be added to the apparent altitude.

Planet's Ap. Alt.	Planet's Horizontal Parallax (see *Nautical Almanac*).																
	″ 1	″ 3	″ 5	″ 7	″ 9	″ 11	″ 13	″ 15	″ 17	″ 19	″ 21	″ 23	″ 25	″ 27	″ 29	″ 31	″ 33
°	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″	″
5	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33
6	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33
7	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33
8	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33
9	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33
10	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33
11	1	3	5	7	9	11	13	15	17	19	21	23	25	27	28	30	32
12	1	3	5	7	9	11	13	15	17	19	21	23	24	26	28	30	32
14	1	3	5	7	9	11	13	15	17	18	20	22	24	26	28	30	32
16	1	3	5	7	9	11	13	14	16	18	20	22	24	26	28	30	32
18	1	3	5	7	9	10	12	14	16	18	20	22	24	26	28	29	31
20	1	3	5	7	8	10	12	14	16	18	20	22	23	25	27	29	31
23	1	3	5	7	8	10	12	14	16	17	19	21	23	25	27	29	30
26	1	3	5	6	8	10	12	13	15	17	19	21	22	24	26	28	30
29	1	3	4	6	8	10	11	13	15	17	18	20	22	24	25	27	29
32	1	3	4	6	8	9	11	13	14	16	18	20	21	23	25	26	28
35	1	3	4	6	7	9	11	12	14	16	17	19	20	22	24	25	27
40	1	2	4	5	7	8	10	12	13	15	16	18	19	21	22	24	25
45	1	2	4	5	6	8	9	11	12	13	15	16	18	19	21	22	23
50	1	2	3	5	6	7	8	10	11	12	14	15	16	17	19	20	21
55	1	2	3	4	5	6	7	9	10	11	12	13	14	15	17	18	19
60	0	1	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
65	0	1	2	3	4	5	6	6	8	8	9	10	11	11	12	13	14
70	0	1	2	2	3	4	4	5	6	7	7	8	9	9	10	11	11
75	0	1	1	2	2	3	3	4	4	5	5	6	6	7	8	8	9
80	0	0	1	1	2	2	2	3	3	3	4	4	4	5	5	5	6
85	0	0	0	0	1	1	1	1	2	2	2	2	2	2	3	3	3

Note.—Sin. hor. par. × cos. alt. = Sin. par. in alt. The horizontal parallax of the moon can be found by this formula.

TABLE VII.—*Augmentation of the Moon's Semi-Diameter.*

To be added to semi-diameter as given in the *Nautical Almanac.*

Moon's App. Altitude.	Moon's Horizontal Semi-diameter.						Moon's App. Altitude.	Moon's Horizontal Semi-diameter.					
	′ ″ 14·30	′ ″ 15·0	′ ″ 15·30	′ ″ 16·0	′ ″ 16·30	′ ″ 17·0		′ ″ 14·30	′ ″ 15·0	′ ″ 15·30	′ ″ 16·0	′ ″ 16·30	′ ″ 17 0
°	″	″	″	″	″	″	°	″	″	″	″	″	″
10	2·5	2·5	3·	3·	3·	3·5	55	11.	12·	13·	13·5	14·5	15·5
15	3·5	4·	4·	4·5	4·5	5·	60	12·	12·5	13·5	14·5	15·5	16·5
20	4·5	5·	5·5	6·	6·	6·5	65	12·5	13·5	14·	15·	16·	17·
25	6·	6·	6·5	7·	7·5	8·	70	13·	14·	14·5	15·5	16·5	17·5
30	7·	7·5	8·	8·5	9·	9·5	75	13·	14·	15·	16·	17·	18·
35	8·	8·5	9·	9·5	10·	11·	80	13·5	14·5	15·5	16·5	17·5	18·5
40	9·	9·5	10·	10·5	11·5	12·	85	13·5	14·5	15·5	16·5	17·5	18·5
45	9·5	10·5	11·	12·	12·5	13·5	90	13·5	14·5	15·5	16·5	17·5	19·
50	10·5	11·	12·	13·	13·5	14·5							

TABLE VIII.—*Correction of Moon's Equatorial Parallax.*

To be subtracted from moon's hor. parallax as given in the *Nautical Almanac.*

Latitude.	Moon's Horizontal Parallax.			Latitude.	Moon's Horizontal Parallax.		
	′ 53	′ 57	′ 61		′ 53	′ 57	′ 61
°	″	″	″	°	″	″	″
10	0·3	0·3	0·4	55	7·	7·7	8·2
15	0·7	0·8	0·8	60	8·	8·5	9·2
20	1·2	1·3	1·4	65	8·7	9·3	10·
25	1·9	2·	2·2	70	9·4	10·	11·
30	2·6	3·	3·	75	9·9	10·7	11·4
35	3·4	3·7	4·	80	10·3	11·	12·
40	4·4	4·7	5·	85	10·5	11·3	12·
45	5·3	5·7	6·	90	10·6	11·4	12·
50	6·2	6·7	7·2				

TABLE IX.—*For Accelerating Mean Solar Time to Convert it into Sidereal Time.*

Reduction to be added to mean solar time.

Hours.		Minutes.				Seconds.			
Hours.	Reduction.	Minutes.	Reduction.	Minutes.	Reduction	Seconds.	Reduction.	Seconds.	Reduction.
	Min. Sec.		Sec.		Sec.		Sec.		Sec.
1	0 9·86	1	0·16	31	5·09	1	0·00	31	0·08
2	0 19·71	2	0·33	32	5·26	2	0·01	32	0·09
3	0 29·57	3	0·49	33	5·42	3	0·01	33	0·09
4	0 39·43	4	0·66	34	5·59	4	0·01	34	0·09
5	0 49·28	5	0·82	35	5·75	5	0·01	35	0·10
6	0 59·14	6	0·99	36	5·91	6	0·02	36	0·10
7	1 9·00	7	1·15	37	6·08	7	0·02	37	0·10
8	1 18·85	8	1·31	38	6·24	8	0·02	38	0·10
9	1 28·71	9	1·48	39	6·41	9	0·02	39	0·11
10	1 38·56	10	1·64	40	6·57	10	0·03	40	0·11
11	1 48·42	11	1·81	41	6·74	11	0·03	41	0·11
12	1 58·28	12	1·97	42	6·90	12	0·03	42	0·11
13	2 8·13	13	2·14	43	7·06	13	0·04	43	0·12
14	2 17·99	14	2·30	44	7·23	14	0·04	44	0·12
15	2 27·85	15	2·46	45	7·39	15	0·04	45	0·12
16	2 37·70	16	2·63	46	7·56	16	0·04	46	0·13
17	2 47·56	17	2·79	47	7·72	17	0·05	47	0·13
18	2 57·42	18	2·96	48	7·89	18	0·05	48	0·13
19	3 7·27	19	3·12	49	8·05	19	0·05	49	0·13
20	3 17·13	20	3·29	50	8·21	20	0·05	50	0·14
21	3 26·99	21	3·45	51	8·38	21	0·06	51	0·14
22	3 36·84	22	3·61	52	8·54	22	0·06	52	0·14
23	3 46·70	23	3·78	53	8·71	23	0·06	53	0·15
24	3 56·56	24	3·94	54	8·87	24	0·07	54	0·15
		25	4·11	55	9·04	25	0·07	55	0·15
		26	4·27	56	9·20	26	0·07	56	0·15
		27	4·44	57	9·36	27	0·07	57	0·16
		28	4·60	58	9·53	28	0·08	58	0·16
		29	4·76	59	9·69	29	0·08	59	0·16
		30	4·93	60	9·86	30	0·08	60	0·16

Example.—Required the sidereal time corresponding to 4h. 18m. 51s. mean solar time.

		H. M. S.
Mean time		4 18 51
Acceleration for	4 hours .	39·43
	18 min. . .	2·96
	51 sec. . .	·14
Sidereal equivalent .		4 19 33·53

TABLE X.—*Conversion of Degrees, &c., of Arc into Time, and vice versâ.*

Arc into Time.						Time into Arc.							
°	Hours. Minutes.	′	Minutes. Seconds.	″	Seconds. Tenths.	Hours.	°	Minutes.	° ′	Seconds.	′ ″	Tenths.	″
1	0 4	1	0 4	1	0·1	1	15	1	0 15	1	0 15	0·1	1·5
2	0 8	2	0 8	2	0·1	2	30	2	0 30	2	0 30	0·2	3·0
3	0 12	3	0 12	3	0·2	3	45	3	0 45	3	0 45	0·3	4·5
4	0 16	4	0 16	4	0·3	4	60	4	1 0	4	1 0	0·4	6·0
5	0 20	5	0 20	5	0·3	5	75	5	1 15	5	1 15	0·5	7·5
6	0 24	6	0 24	6	0·4	6	90	6	1 30	6	1 30	0·6	9·0
7	0 28	7	0 28	7	0·5	7	105	7	1 45	7	1 45	0·7	10·5
8	0 32	8	0 32	8	0·5	8	120	8	2 0	8	2 0	0·8	12·0
9	0 36	9	0 36	9	0·6	9	135	9	2 15	9	2 15	0·9	13·5
10	0 40	10	0 40	10	0·7	10	150	10	2 30	10	2 30		
11	0 44	11	0 44	11	0·7	11	165	11	2 45	11	2 45		
12	0 48	12	0 48	12	0·8	12	180	12	3 0	12	3 0		
13	0 52	13	0 52	13	0·9	13	195	13	3 15	13	3 15		
14	0 56	14	0 56	14	0·9	14	210	14	3 30	14	3 30		
15	1 0	15	1 0	15	1·0	15	225	15	3 45	15	3 45		
16	1 4	16	1 4	16	1·1	16	240	16	4 0	16	4 0		
17	1 8	17	1 8	17	1·1	17	255	17	4 15	17	4 15		
18	1 12	18	1 12	18	1·2	18	270	18	4 30	18	4 30		
19	1 16	19	1 16	19	1·3	19	285	19	4 45	19	4 45		
20	1 20	20	1 20	20	1·3	20	300	20	5 0	20	5 0		
21	1 24	21	1 24	21	1·4	21	315	21	5 15	21	5 15		
22	1 28	22	1 28	22	1·5	22	330	22	5 30	22	5 30		
23	1 32	23	1 32	23	1·5	23	345	23	5 45	23	5 45		
24	1 36	24	1 36	24	1·6	24	360	24	6 0	24	6 0		
25	1 40	25	1 40	25	1·7			25	6 15	25	6 15		
26	1 44	26	1 44	26	1·7			26	6 30	26	6 30		
27	1 48	27	1 48	27	1·8			27	6 45	27	6 45		
28	1 52	28	1 52	28	1·9			28	7 0	28	7 0		
29	1 56	29	1 56	29	1·9			29	7 15	29	7 15		
30	2 0	30	2 0	30	2·0			30	7 30	30	7 30		
31	2 4	31	2 4	31	2·1			31	7 45	31	7 45		
32	2 8	32	2 8	32	2·1			32	8 0	32	8 0		
33	2 12	33	2 12	33	2·2			33	8 15	33	8 15		
34	2 16	34	2 16	34	2·3			34	8 30	34	8 30		
35	2 20	35	2 20	35	2·3			35	8 45	35	8 45		
36	2 24	36	2 24	36	2·4			36	9 0	36	9 0		
37	2 28	37	2 28	37	2·5			37	9 15	37	9 15		
38	2 32	38	2 32	38	2·5			38	9 30	38	9 30		
39	2 36	39	2 36	39	2·6			39	9 45	39	9 45		
40	2 40	40	2 40	40	2·7			40	10 0	40	10 0		
50	3 20	41	2 44	41	2·7			41	10 15	41	10 15		
60	4 0	42	2 48	42	2·8			42	10 30	42	10 30		
70	4 40	43	2 52	43	2·9			43	10 45	43	10 45		
80	5 20	44	2 56	44	2·9			44	11 0	44	11 0		
90	6 0	45	3 0	45	3·0			45	11 15	45	11 15		
100	6 40	46	3 4	46	3·1			46	11 30	46	11 30		
110	7 20	47	3 8	47	3·1			47	11 45	47	11 45		
120	8 0	48	3 12	48	3·2			48	12 0	48	12 0		
130	8 40	49	3 16	49	3·3			49	12 15	49	12 15		
140	9 20	50	3 20	50	3·3			50	12 30	50	12 30		
150	10 0	51	3 24	51	3·4			51	12 45	51	12 45		
160	10 40	52	3 28	52	3·5			52	13 0	52	13 0		
170	11 20	53	3 32	53	3·5			53	13 15	53	13 15		
180	12 0	54	3 36	54	3·6			54	13 30	54	13 30		
		55	3 40	55	3·7			55	13 45	55	13 45		
		56	3 44	56	3·7			56	14 0	56	14 0		
		57	3 48	57	3·8			57	14 15	57	14 15		
		58	3 52	58	3·9			58	14 30	58	14 30		
		59	3 56	59	3·9			59	14 45	59	14 45		

Table XI.—*For Reducing Circum-Meridian Observations to the Meridian.*

(See Example 4, p. 43.)

Seconds.	Hour Angle.																					
	0 Min.	Diff. for 1 Sec.	1 Min.	Diff. for 1 Sec.	2 Min.	Diff. for 1 Sec.	3 Min.	Diff. for 1 Sec.	4 Min.	Diff. for 1 Sec.	5 Min.	Diff. for 1 Sec.	6 Min.	Diff. for 1 Sec.	7 Min.	Diff. for 1 Sec.	8 Min.	Diff. for 1 Sec.	9 Min.	Diff. for 1 Sec.	10 Min.	Diff. for 1 Sec.
0	0·0	·0	2·0	·1	7·8	·1	17·7	·2	31·4	·3	49·1	·3	70·7	·4	96·2	·5	125·6	·5	159·0	·6	196·3	·7
10	0·0	·0	2·7	·1	9·2	·1	19·7	·2	34·1	·3	52·4	·3	74·7	·4	100·8	·5	130·9	·5	165·0	·6	202·9	·7
20	0·2	·0	3·5	·1	10·7	·2	21·8	·2	36·9	·3	55·8	·4	78·7	·4	105·6	·5	136·3	·5	171·0	·6	209·7	·7
30	0·5	·0	4·4	·1	12·3	·2	24·0	·2	39·8	·3	59·4	·4	82·9	·4	110·4	·5	141·8	·6	177·2	·6	216·4	·7
40	0·9	·0	5·4	·1	14·0	·2	26·4	·2	42·8	·3	63·0	·4	87·3	·4	115·4	·5	147·5	·6	183·5	·6	223·4	·7
50	1·4	·1	6·6	·1	15·8	·2	28·8	·3	45·9	·3	66·8	·4	91·7	·4	120·5	·5	153·2	·6	189·8	·6	230·4	·7

[*Continued.*

TABLE XI.—(*Continued*).

HOUR ANGLE.

Seconds.	11 Min.	Diff. for 1 Sec.	12 Min.	Diff. for 1 Sec.	13 Min.	Diff. for 1 Sec.	14 Min.	Diff. for 1 Sec.	15 Min.	Diff. for 1 Sec.	16 Min.	Diff. for 1 Sec.	17 Min.	Diff. for 1 Sec.	18 Min.	Diff. for 1 Sec.	19 Min.	Diff. for 1 Sec.	20 Min.	Diff. for 1 Sec.
0	237·5	·7	282·7	·8	331·7	·9	384·7	·9	441·6	1·0	502·5	1·0	567·2	1·1	635·9	1·2	708·4	1·2	784·9	1·3
10	244·8	·7	290·6	·8	340·3	·9	393·9	·9	451·5	1·0	513·0	1·1	578·4	1·1	647·7	1·2	720·9	1·3	798·0	1·3
20	252·1	·7	298·6	·8	349·0	·9	403·3	·9	461·5	1·0	523·6	1·1	589·6	1·1	659·6	1·2	733·5	1·3	811·3	1·3
30	259·6	·8	306·7	·8	357·7	·9	412·7	·9	471·5	1·0	534·3	1·1	601·0	1·1	671·6	1·2	745·2	1·3	824·6	1·3
40	267·2	·8	315·0	·8	366·6	·9	422·2	1·0	481·7	1·0	545·2	1·1	612·5	1·2	683·8	1·2	759·0	1·3	838·0	1·4
50	274·9	·8	323·3	·8	375·6	·9	431·9	1·0	492·0	1·0	556·1	1·1	624·1	1·2	696·0	1·2	771·9	1·3	851·6	

Example.—Required the number corresponding to the hour angle 13 min. 33 sec.

The next less hour angle given in the table =	13 min. 30 sec.		No. 357·7
Difference for 1 sec. ,, ,, ,, = ·9	3 ,,	× ·9 =	2·7
Number corresponding to	13 min. 33 sec.	=	360·4